Name of the proposed cryptosystem

LEDAkem (Low dEnsity coDe-bAsed key encapsulation mechanism)

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LEDAkem: Low dEnsity coDe-bAsed key encapsulation mechanism

Specification revision 1.0 – November 30, 2017

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Chapter 1

Complete written specification

LEDAkem is a Key Encapsulation Module (KEM) built from the Niederreiter cryptosystem based on linear error-correcting codes. In particular, LEDAkem exploits the advantages of relying on Quasi-Cyclic Low-Density Parity-Check (QC-LDPC) codes providing high decoding speeds and compact keypairs [4, 5], with two main innovations:

i. The use of ephemeral keys is employed to foil statistical attacks such as the one reported in [14].

ii. A new decoding algorithm is designed: it provides faster decoding than the regular bit-flipping decoding procedure, saves a computationally demanding matrix inverse computation, and allows a reduction in the required private key storage.

The main known attacks against this system are those applicable against QC-LDPC code-based cryptosystems [4], which have been studied for ten years since the first proposal appeared in [3], plus statistical attacks recently introduced in [14,18]. We carefully analyze their capabilities and provide parametrization for the LEDAkem system to provide the required security guarantees taking into account the computational cost reduction of solving the underlying computationally hard problem caused by such attacks.

1.1 Preliminaries

We now provide a set of background notions and nomenclature concerning binary error correcting codes, and in particular Low-Density Parity-Check codes, which are the foundational constructs of LEDAkem.

1.1.1 Linear error correcting codes

Binary error correcting codes rely on a redundant representation of information in the form of binary strings to be able to detect and/or correct accidental bit errors which may happen during transmission or storage. We will employ binary codes acting on a finite binary sequence at once, known as the information word, which are known as block codes. We will refer to them from now

on simply as binary codes.
In this setting, let $\mathbb{F}_2$ be the binary finite field with the addition and multiplication operations which corresponds to the usual exclusive-or and logical product between two Boolean values. Let $\mathbb{F}_2^k$ denote the k-dimensional vector space defined on $\mathbb{F}_2$. A binary code, denoted as $\mathcal{C}(n,k)$, is defined as a bijective map $\mathcal{C}(n,k) : \mathbb{F}_2^k \to \mathbb{F}_2^n$, $n,k \in \mathbb{N}$, $0 < k < n$, between any binary k-tuple (i.e., an information word) and a binary n-tuple (denoted as codeword). The value n is known as the length of the code, while k is denoted as its dimension.
Encoding through $\mathcal{C}(n,k)$ means converting an information word $u \in \mathbb{F}_2^k$ into its corresponding codeword $c \in \mathbb{F}_2^k$. The decoding process, instead, given a codeword $\hat{c}$ corrupted by an error vector $e \in \mathbb{F}_2^k$ with Hamming weight $t > 0$ ($\hat{c} = c + e$), recovers both the value of the information word u and the value of the error vector e. A code is said to be t-error correcting if, for any value of e, given $\tilde{c}$ there is a decoding procedure to retrieve both the error vector e and the original information word u.

Definition 1.1.1 (Linear Code) The code $\mathcal{C}(n,k)$ is linear if and only if the set of its 2^k codewords is a k-dimensional subspace of the vector space $\mathbb{F}_2^n$.

A property of linear block codes that follows from Definition 1.1.1 is that the sum modulo 2, i.e., the component wise exclusive-or, of two codewords is also a codeword.

Definition 1.1.2 (Minimum distance) Given a linear binary code $\mathcal{C}(n,k)$, the minimum distance of $\mathcal{C}(n,k)$ is the minimum Hamming distance among all the ones which can be computed between a pair of its codewords.

If the code is linear, its minimum distance coincides with the minimum Hamming weight among the ones of its codewords. Given $\mathcal{C}(n,k)$, a linear error correcting code, and $\Gamma \subset \mathbb{F}_2^n$ the vector subspace containing its 2^k codewords, it is possible to represent it choosing k linearly independent codewords $\{g_0, g_1, \ldots g_{k-1}\} \in \mathbb{F}_2^n$ to form a basis of Γ. Any codeword $c = [c_0, c_1, \ldots, c_{n-1}]$ can be expressed as a linear combination of the vectors of the basis:

$$c = u_0 g_0 + \mathrm{u}_1 g_1 + \ldots + u_{k-1} g_{k-1} \tag{1.1}$$

where the binary coefficients u_i can be thought as the element of an information vector $u = [u_0, u_1, \ldots, u_{k-1}]$, which the code maps into c. Equation (1.1) can be rewritten as $c = uG$, where G is a $k \times n$ binary matrix known as the generator matrix of the code $\mathcal{C}(n,k)$, i.e.:

$$G = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{k-1} \end{bmatrix}$$

Since any set of k linearly independent codewords can be used to form G, a code can be represented by different generator matrices. Among the possible representations, it is always possible for a linear code to derive a representation known as systematic.

Definition 1.1.3 (Systematic Code) A linear error correcting code $\mathcal{C}(n,k)$ is said to be in a systematic form, or systematic in short, if each one of its codewords contains the information vector it is associated to.

A conventional way to express a systematic code is the one where each n-bit codeword, c, is obtained by appending $r = n - k$ redundancy bits $(c_k, c_{k+1}, \ldots, c_{n-1})$ to its corresponding k-bit information word (i.e., $c_0, c_1, \ldots, c_{k-1}$, with $c_i = u_i$, $0 \leq i < k$): $c = [u_0, u_1, \ldots, u_{k-1} | c_k, c_{k+1}, \ldots, c_{n-1}]$. It follows that the associated $k \times n$ generator matrix G can be written as $G = [I_k | P]$, where I_k denotes the $k \times k$ identity matrix and P is a $k \times r$ binary matrix.
Let us consider the set of all n-bit vectors in $\mathbb{F}_2^n$ that are orthogonal to any codeword of the code subspace Γ, known as its orthogonal complement $\Gamma^{\perp}$. Its dimension is $\dim\left(\Gamma^{\perp}\right) = n - \dim(\Gamma) = n - k = r$. A basis of $\Gamma^{\perp}$ is readily obtained choosing r linearly independent vector in $\Gamma^{\perp}$ as

$$H = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{r-1} \end{bmatrix}$$

The $r \times n$ matrix H is known as a parity-check matrix of the code $\mathcal{C}(n, k)$, while, for any n-bit vector $x \in \mathbb{F}_2^n$, the $r \times 1$ vector $s = Hx^T$ is known as the syndrome of x through H. Given that H is a basis of $\Gamma^{\perp}$, every codeword $c \in \Gamma$ satisfies the equality $Hc^T = 0_{r \times 1}$ where $0_{r \times 1}$ is the $r \times 1$ zero vector, i.e., a codeword belonging to $\mathcal{C}(n, k)$ has a null syndrome through H.

It can be shown that the generator matrix G and the parity-check matrix H are two equivalent descriptions of a linear code. Indeed, we have that $Hc^T = HG^T u^T = 0_{r \times 1}$, $\forall\, u \in \mathbb{F}_2^k$, yielding in turn that $HG^T = 0_{r \times k}$. Exploiting the aforementioned relation, it is possible to derive H from G and vice-versa. Consider, for the sake of clarity, the case of a systematic code $\mathcal{C}(n, k)$ with $G = [I_k | P]$. It is possible to obtain the corresponding parity-check matrix H as $\left[P^T | I_r\right]$, where ^{T} denotes transposition, which satisfies $HG^T = P^T + P^T = 0_{r \times k}$. Finally, considering a generic parity-check matrix $H = [A|B]$, with A an $r \times k$ matrix and B an $r \times r$ non-singular matrix, a systematic generator matrix of the corresponding code is computed as $G = \left[I_k | \left(B^{-1}A\right)^T\right]$.

1.1.2 Quasi-Cyclic codes and circulant matrices

A Quasi-Cyclic (QC) code is defined as a linear block code $\mathcal{C}(n, k)$ having information word size $k = pk_0$ and codeword size $n = pn_0$, where n_0 is denoted as *basic block length* of the code and each cyclic shift of a codeword by n_0 symbols results in another valid codeword [38].
LEDAkem hinges on a QC code $\mathcal{C}(pn_0, pk_0)$ having the generator and parity-check matrices composed by $p \times p$ circulant sub-matrices (blocks).

A $v \times v$ circulant matrix A has the following form:

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{v-1} \\ a_{v-1} & a_0 & a_1 & \cdots & a_{v-2} \\ a_{v-2} & a_{v-1} & a_0 & \cdots & a_{v-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & a_3 & \cdots & a_0 \end{bmatrix} \tag{1.2}$$

According to its definition, any circulant matrix has a constant row and column weight, i.e., is *regular*, since all its rows and columns are cyclic shifts of the first row and column, respectively.

The set of $v \times v$ binary circulant matrices forms an algebraic ring under the standard operations of modulo-2 matrix addition and multiplication. The zero element is the all-zero matrix, and the

identity element is the $v \times v$ identity matrix. The algebra of the polynomial ring $\mathbb{F}_2[x]/\langle x^v + 1\rangle$ is isomorphic to the ring of $v \times v$ circulant matrices over $\mathbb{F}_2$ with the following map:

$$A \leftrightarrow a(x) = \sum_{i=0}^{v-1} a_i x^i \tag{1.3}$$

According to (1.3), any binary circulant matrix is associated to a polynomial in the variable x having coefficients over $\mathbb{F}_2$ which coincide with the entries in the first row of the matrix

$$a(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_{v-1} x^{v-1} \tag{1.4}$$

In addition, according to eq. (1.3), the all-zero circulant matrix corresponds to the null polynomial and the identity matrix to the unitary polynomial.

The ring of polynomials $\mathbb{F}_2[x]/\langle x^v + 1\rangle$ includes elements that are zero divisors: such elements are mapped onto singular circulant matrices over $\mathbb{F}_2$. Avoiding such matrices is important in the computation of the LEDAkem primitives, as computing the inverse of $v \times v$ circulant matrices is required. However, a proper selection of the size of a circulant matrix v, allows to easily generate invertible instances of it as described in the following.

1.1.3 Polynomial inversion in a finite field

To provide efficient execution for the LEDAkem primitives, it is crucial to be able to efficiently check invertibility of a binary circulant matrix, and to generate a non-singular circulant matrix efficiently. To this end, we exploit the isomorphism (1.3) between $p \times p$ binary circulant matrices and polynomials in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$, turning the problem into providing an efficient criterion for the invertibility of an element of $\mathbb{F}_2[x]/\langle x^p + 1\rangle$ and providing an efficient way to generate such invertible polynomials. In the following, we recall some facts from finite field theory, and we derive a necessary and sufficient condition for the invertibility of an element of $\mathbb{F}_2[x]/\langle x^p + 1\rangle$, provided p is chosen according to the described criterion. Let $\mathbb{F}_{q^m}$ be a finite field, with q a prime power and m a positive integer; given an element $\alpha \in \mathbb{F}_{q^m}$, the following propositions hold [39]:

(i) The minimal polynomial of α with respect to $\mathbb{F}_q$, i.e., the nonzero monic polynomial $f(x) \in \mathbb{F}_q[x]$ of the least degree such that $f(\alpha) = 0$, always exists, it is unique, and it is also irreducible over $\mathbb{F}_q$.

(ii) If a monic irreducible polynomial $g(x) \in \mathbb{F}_q[x]$ has $\alpha \in \mathbb{F}_{q^m}$ as a root, then it is the minimal polynomial of α with respect to $\mathbb{F}_q$.

Definition 1.1.4 Let n be a positive integer and q a prime power such that $\texttt{gcd}(n, q) = 1$. A cyclotomic coset of q modulo n containing the value $a \in \mathbb{Z}_n$ is defined as

$$C_a = \{aq^j \bmod n : j = 0, 1, \ldots\}$$

A subset $\{a_1, \ldots, a_s\} \subseteq \mathbb{Z}_n$ is named as a *complete set of representatives* of cyclotomic cosets of q modulo n if $\forall\, i \neq j\ C_{a_i} \cap C_{a_j} = \varnothing$ and $\bigcup_j^s C_{a_j} = \mathbb{Z}_n$.

It is worth noting that the previous definition allows to easily infer that two cyclotomic cosets are either equal or disjoint. Indeed, given two cyclotomic cosets C_{a_1} and C_{a_2}, with $a_1 \neq a_2 \bmod n$, if $C_{a_1} \cap C_{a_2} \neq \varnothing$, two positive integers j and k such that $a_1 q^j = a_2 q^k \bmod n$ should exist. Assuming (without loss of generality) that $k \geq j$, the condition $\texttt{gcd}(n, q) = 1$ would ensure the existence of the multiplicative inverse of q and consequentially that $a_1 = a_2 q^{k-j} \bmod n$, which in turn would imply that the cyclotomic coset including a_1 is a subset of the coset including a_2, i.e., $C_{a_1} \subseteq C_{a_2}$. However, as the previous equality can be rewritten as $a_2 = a_1 (q^{-1})^{k-j} \bmod n$, it would also imply $C_{a_2} \subseteq C_{a_1}$, leading to conclude that $a_1 = a_2 \bmod n$, which is a contradiction of the initial assumption about them being different.

Two notable theorems that make use of the cyclotomic coset definition to determine the minimal polynomials of every element in a finite field can be stated as follows [39].

Theorem 1.1.1 Let α be a primitive element of $\mathbb{F}_{q^m}$, the minimal polynomial of α^i in $\mathbb{F}_q[x]$ is $g^{(i)}(x) = \prod_{j \in C_i} (x - \alpha^j)$, where C_i is the unique cyclotomic coset of q modulo $q^m - 1$ containing i.

Theorem 1.1.2 Given a positive integer n and a prime power q, with $\texttt{gcd}(n, q) = 1$, let m be a positive integer such that $n \,|\, (q^m - 1)$. Let α be a primitive element of $\mathbb{F}_{q^m}$ and let $g^{(i)}(x) \in \mathbb{F}_q[x]$ be the minimal polynomial of $\alpha^i \in \mathbb{F}_{q^m}$. Denoting as $\{a_1, \ldots, a_s\} \subseteq \mathbb{Z}_n$ a complete set of representatives of cyclotomic cosets of q modulo n, the polynomial $x^n - 1 \in \mathbb{F}_q[x]$ can be factored as the product of monic irreducible polynomials over $\mathbb{F}_q$:

$$x^n - 1 = \prod_{i=1}^{s} g^{\left(\frac{(q^m-1)a_i}{n}\right)}(x)$$

Corollary 1.1.1 Given a positive integer n and a prime power q, with $\texttt{gcd}(n, q) = 1$, the number of monic irreducible factors of $x^n - 1 \in \mathbb{F}_q[x]$ is equal to the number of cyclotomic cosets of q modulo n.

From the previous propositions on the properties of finite fields, it is possible to derive the following results:

Corollary 1.1.2 Given an odd prime number p, if 2 is a primitive element in the finite field $\mathbb{Z}_p$ then the irreducible (non trivial) polynomials being a factor of $x^p - 1 \in \mathbb{F}_2[x]$ are $x + 1$ and $\Phi(x) = x^{p-1} + x^{p-2} + \cdots + x + 1$.

Proof. Considering the ring of polynomials with binary coefficients $\mathbb{F}_2[x]$ and picking a positive integer n as an odd prime number (i.e., $n = p$), Corollary 1.1.1 ensures that the number of factors of $x^p - 1 \in \mathbb{F}_2[x]$ equals the number of cyclotomic cosets of 2 modulo p.
If 2 is a primitive element of $\mathbb{Z}_p$, its order, $\texttt{ord}_p(2)$, is equal to the order of the (cyclic) multiplicative group of the field, i.e., $\texttt{ord}_p(2) = |\,(\mathbb{Z}_p \setminus \{0\}, \cdot)\,| = p - 1$ thus, the said cyclotomic cosets can be listed as: $C_0 = \{0 \cdot 2^j \bmod p : j = 0, 1, \ldots\} = \{0\}$ and $C_1 = \{1 \cdot 2^j \bmod p : j = 0, 1, \ldots\} = \mathbb{Z}_p \setminus \{0\}$. The polynomial $x^p - 1 \in \mathbb{F}_2[x]$ admits $\alpha = 1$ as a root, therefore its two (non trivial) factors can be listed as: $x - 1$ and $\frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \cdots + x + 1$. ■

Theorem 1.1.3 (Invertible elements in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$) Let p be a prime number such that $\texttt{ord}_p(2) = p - 1$. Let $g(x)$ be a binary polynomial in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$, with $\texttt{deg}(g(x)) > 0$.
$g(x)$ has a multiplicative inverse in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$ if and only if it contains an odd number of terms and $g(x) \neq \Phi(x)$, with $\Phi(x) = x^{p-1} + x^{p-2} + \cdots + x + 1$.

Proof. If $g(x) \in \mathbb{F}_2[x]/\langle x^p+1\rangle$ contains an odd number of terms and $g(x) \neq \Phi(x)$, to prove it is invertible modulo x^p+1 we need to consider that $\texttt{gcd}(g(x), x^p+1) = \texttt{gcd}(g(x), (x+1)\Phi(x))$.
It is easy to observe that $x+1$ does not divide $g(x)$, i.e., $(x+1) \nmid g(x)$, as $g(1) = 1$, thus they are coprime. Considering $\Phi(x)$, we know by hypothesis that $\texttt{ord}_p(2) = p-1$, therefore $\Phi(x)$ is irreducible over $\mathbb{F}_2[x]$ (see Corollary 1.1.2), which excludes that $g(x) \mid \Phi(x)$.
To the end of proving $g(x)$ and $\Phi(x)$ coprime, it has to hold that $\Phi(x) \nmid g(x)$. To this end assume, by contradiction, that $g(x)h(x) = \Phi(x)$ for a proper choice of $h(x) \in \mathbb{F}_2[x]$. The previous equality entails that $\texttt{deg}(g(x)) + \texttt{deg}(h(x)) = p-1$, while $\texttt{deg}(g(x)) \leq p-1$, which in turn leaves $\texttt{deg}(h(x)) = 0$ as the only option, leading to conclude $h(x) = 0$ or $h(x) = 1$. In case $h(x) = 0$, the equality $g(x) \cdot 0 = x^{p-1} + x^{p-2} + \cdots + x + 1$ is false, while in case $h(x) = 1$, the equality $g(x) \cdot 1 = \Phi(x)$ contradicts the hypothesis. Since we proved that $g(x) \nmid \Phi(x)$ and $\Phi(x) \nmid g(x)$, $g(x) \neq \Phi(x)$ by hypothesis, we can infer that they are coprime.
Finally, being $\texttt{gcd}(g(x), x^p+1) = \texttt{gcd}(g(x), (x+1)\Phi(x)) = 1$ we conclude that g(x) is invertible.
To prove the other implication of the theorem, if $g(x) \in \mathbb{F}_2[x]/\langle x^p+1\rangle$, with degree $\texttt{deg}(g(x)) > 0$ is invertible we need to derive that $g(x)$ must have an odd number of terms and be different from $\Phi(x)$. Being $g(x)$ invertible, this means that $\texttt{gcd}(g(x), x^p+1) = \texttt{gcd}(g(x), (x+1)\Phi(x)) = 1$, which in turn means that $\texttt{gcd}(g(x), x+1) = 1$ and $\texttt{gcd}(g(x), \Phi(x)) = 1$ that guarantees that $g(x) \neq \Phi(x)$ and that $g(1) = 1$. Willing to prove that $g(x)$ must have an odd number of terms, assume, by contradiction, it has an even number of terms. Regardless of which terms are contained in $g(x)$ this means that it admits 1 as a root, which contradicts the premise. ■

1.1.4 Quasi-Cyclic Low-Density Parity-Check codes and their efficient decoding

A Low-Density Parity-Check (LDPC) code $\mathcal{C}(n,k)$ is a special type of linear block code characterized by a sparse parity-check matrix H. In particular, the Hamming weight of a column of H, denoted as d_v, is much smaller than its column length r and increases sub-linearly with it. In terms of error correction capability, LDPC codes having a non-constant weight for either the rows or the columns of H, hence known as irregular LDPC codes, were proven to approach the channel capacity [24]. Considering the parity-check matrix H of an LDPC code as the incidence matrix of a graph, such a graph is known as Tanner graph, and it has been shown that the presence of a small number of short cycles in it is beneficial to the error correction performance of the code.

The peculiar form of LDPC codes allows to devise an efficient decoding procedure, provided their parity-check matrix H is known, via algorithms known as Bit Flipping (BF) decoders [16]. Indeed, BF algorithms perform decoding with a fixed-point procedure which exploits the form of H to iteratively deduce which bits of an error-affected codeword should be flipped in order to obtain a zero-valued syndrome for it. If the fixed-point procedure converges within a desired amount of iterations to a zero-valued syndrome, the decoding action is deemed successful.

The main observation of the BF decoders starts from considering the parity-check matrix H as the description of a set of r equations in the codeword bits yielding the syndrome bits as their results. Such equations are known as parity-check equations, or parity checks, in short. In this context, the one-valued coefficients of the i-th column of a parity-check matrix H can be thought of as the indicators of which parity checks of the code are involving the i-th bit of the received codeword. The result of each one of the said parity checks is a bit in the syndrome, hence a zero-valued syndrome indicates a set of successful parity checks, and thus a correct codeword. The convergence of the fixed-point decoder is influenced by the number of parity checks in which each codeword entry is involved: in particular, being involved in a small number of parity checks speeds up the convergence.

Algorithm 1.1.1: BF decoding

Input: x: QC-LDPC error-affected codeword as a $1 \times pn_0$ binary vector.
s: QC-LDPC syndrome. It is a $pr_0 \times 1$ binary vector obtained as $s = Hx^T$.
`Hsparse`: sparse version of the parity-check matrix H, represented as an $d_v \times n_0$ integer matrix containing for each of its n_0 columns, the positions in $\{0, 1, \ldots, pr_0 - 1\}$ of the asserted binary coefficients in the first column of the sequence of r_0 circulant block matrices (each of which with size $p \times p$).

Output: c: error-free $1 \times pn_0$ codeword
`decodeOk`: Boolean value denoting the successful outcome of the decoding action

Data: `imax`: the maximum number of allowed iterations before reporting a decoding failure

```
1  codeword ← x                                              // bitvector with size pn0
2  syndrome ← s                                              // bitvector with size pr0
3  iterationCounter ← 0     // scalar variable denoting the number of iterations
4  repeat
5     unsatParityChecks ← 0_{1×pr0} // counters of unsatisfied parity checks
6     for i = 0 to pn0 − 1 do
7        for j = 0 to dv − 1 do
8           assertedHbitPosition ← (i + Hsparse[j][i]) mod p + p · ⌊Hsparse[j][i] div p⌋
9           if syndrome[assertedHbitPosition] = 1 then
10             unsatParityChecks[i] ← 1 + unsatParityChecks[i]
11    maxUPC ← MAX(unsatParityChecks)
12    for i = 0 to pn0 − 1 do
13       if unsatParityChecks[i] = maxUPC then
14          BitToggle(codeword[i])                                  // codeword update
15          for j = 0 to dv − 1 do
16             assertedHbitPos ← (i + Hsparse[j][i]) mod p + p · ⌊Hsparse[j][i] div p⌋
17             BitToggle(syndrome[assertedHbitPos])
18 until syndrome ≠ 0_{1×pr0}  AND  iter < imax
19 if syndrome = 0_{1×pr0} then
20    return codeword, true
21 return codeword, false
```

An LDPC code may also be a QC code, expressed with a QC parity-check or generator matrix, hence being named a QC-LDPC code, which is indeed the case of the codes employed in LEDAkem.

An efficient BF decoding procedure for QC-LDPC codes can be devised relying on the number of unsatisfied parity checks to which a codeword bit concurs as an estimate of it being affected by an error. We describe such a procedure in Algorithm 1.1.1, where the sparse and QC nature of the matrix H is explicitly exploited. To this end H is represented as $r_0 \times n_0$ sparse $p \times p$ circulant blocks, and only the positions of the first column of each block are stored in `Hsparse`. Algorithm 1.1.1 receives, alongside `Hsparse`, the error-affected codeword to be corrected x, its syndrome computed as $s = Hx^T$, and performs the fixed-point decoding procedure for a maximum of `imax` iterations. The algorithm outputs its best estimate for the correct codeword c and a boolean variable `decodeOk` reporting the success of the decoding procedure. The procedure iterates at fixed-point (loop at lines 4–18) the decoding procedure, which starts by counting how many

unsatisfied parity checks a codeword bit is involved into (lines 5–10). Such a value is obtained considering which are the asserted bits in a given column of H, taking care of accounting for its sparse representation, and the cyclic nature of its blocks (line 8). Whenever a bit in the i-th column and `assertedHbitPos`-th row of H is set, it is pointing to the fact that the i-th bit of the codeword is involved in the `assertedHbitPos`-th parity-check equation. Thus, if the `assertedHbitPos`-th bit of the syndrome is unsatisfied, i.e., equal to 1, the number of unsatisfied parity checks of the i-th bit is incremented (lines 9–10). Once the computation of the number of unsatisfied parity checks per codeword bit is completed, a decision must be taken on which of them are to be flipped, as they are deemed error affected. A possible criterion, yielding very good error correction performances at the cost of a lower computational efficiency is to flip all the bits which are involved in the maximum number of unsatisfied parity checks, computed at line 11. Thus, the procedure toggles the values of all the codeword bits for which the number of unsatisfied parity checks matches the maximum one (lines 12–14). Once this step is completed, the values of the parity checks should be recomputed according to the new value of the codeword. While this can be accomplished by pre-multiplying the transposed codeword by H, it is more efficient to exploit the knowledge of which bits of the codeword were toggled to change only the parity-check values in the syndrome affected by such toggles. Lines 15–17 of Algorithm 1.1.1 update the syndrome according to the aforementioned procedure, i.e., for a given i-th codeword bit being toggled, all the syndrome values corresponding to the positions of the asserted coefficients in the i-th column of H are also toggled. Once either the decoding procedure has reached its intended fixed-point, i.e., the syndrome is a zero-filled vector, or the maximum number of iterations has been reached, Algorithm 1.1.1 returns its best estimate for the corrected codeword, together with the outcome of the decoding procedure (lines 19–21).

1.2 Niederreiter cryptosystem

The Niederreiter cryptosystem [29] is a code-based cryptosystem exploiting the same trapdoor introduced in the McEliece cryptosystem [26], but under an alternative formulation. The main difference between McEliece and Niederreiter is in that Niederreiter exploits syndromes and parity-check matrices instead of codewords and generator matrices used in McEliece. The original proposal of the Niederreiter cryptosystem used Generalized Reed-Solomon (GRS) codes as private codes, which however have been shown to expose the system to vulnerabilities. Nevertheless, when the same family of codes is used, Niederreiter and McEliece cryptosystems are equivalent [23] and therefore they achieve the same security levels. Key generation, encryption and decryption in the Niederreiter cryptosystem work as follows.

Key generation. In order to generate his key pair, Bob chooses two secret matrices:

i. The $r \times n$ parity-check matrix H of a linear block code able to correct t errors.

ii. A random non-singular $r \times r$ scrambling matrix S.

Bob's private key is $\{H, S\}$, while his public key is computed as

$$H' = SH. \tag{1.5}$$

It is worth observing that the rows of H' are linear combinations of the rows of H, representing the parity-check equations of the private code. Therefore H' defines a public code that coincides with

the private code. However, the code representation through H' does not allow performing decoding through efficient decoding algorithms, and decoding through general algorithms has exponential complexity in the code length.

Encryption. In order to send an encrypted message m to Bob, Alice maps m into one or more n-bit strings with weight t. Then, she uses Bob's public key H' to obtain the encrypted version of each weight-t string e as its syndrome computed through H', that is,

$$x = H'e^T = SHe^T. \tag{1.6}$$

Decryption. In order to decrypt each received message (or part of a message) x, Bob computes the syndrome of e through H as

$$x' = S^{-1}x = He^T, \tag{1.7}$$

then he performs syndrome decoding through the secret code to obtain e from x'. Finally, Bob demaps e into the secret message m, or part of it.

In the original McEliece and Niederreiter cryptosystems, algebraic code families provided with bounded-distance decoders were considered. In such a case, since the number of errors correctable by the secret code is t, it is guaranteed that Bob is able to recover e from x' and the Decryption Failure Rate (DFR) is zero.

1.3 The LEDAkem cryptosystem

The LEDAkem cryptosystem is derived from the Niederreiter cryptosystem with the following main differences:

- Non-algebraic QC-LDPC codes are used as private codes. In particular, LEDAkem hinges on a family of QC-LDPC codes, $\mathcal{C}(n, k)$ with $n = pn_0$, $k = p(n_0 - 1)$, having a (small) basic block length n_0, and a single $p \times p$ redundancy block (i.e., $r = n - k = pr_0 = p$, $r_0 = 1$) in each codeword, where p is an odd prime integer [1, 2].
- The public code is neither coincident with nor equivalent to the private code.
- Suitably designed iterative non-bounded-distance decoding algorithms are used.

The motivation of using QC-LDPC codes as private codes is in the fact that these codes are known to achieve important reductions in the public key size when used in this context [1, 27]. However, when LDPC codes are used as private codes, the public code cannot be either coincident with or equivalent to the private code. Otherwise, an attacker could search for low weight codewords in the dual of the public code and find a sparse parity-check matrix of the private code which allows efficient decoding.

For this reason, following [1], LEDAkem uses a transformation matrix Q that hides the sparse parity-check matrix H of the private code into a denser parity-check matrix $L = HQ$ of the public code. This also affects the error vector that must be corrected during decryption, which is obtained from the error vector used during encryption through multiplication by Q. However, efficient

iterative decoding algorithms are proposed that exploit the knowledge of Q to achieve very good performance in terms of speed and DFR.

In fact, a well-known feature of LDPC coding is that the decoding radius of iterative decoders cannot be estimated in a deterministic way, therefore some residual DFR must be tolerated, and it must be estimated heuristically through Montecarlo simulations. This is done for all the proposed instances of LEDAkem in order to guarantee that they achieve a sufficiently low DFR. Despite the presence of decryption failures, our scheme exhibits the indistinguishability under chosen ciphertext attack (IND-CCA) property, in the sense claimed in [30]. Indeed, the IND-CCA conversion described in [30] works only if there are no decryption failures or the attacker cannot derive from them any information. This is exactly what happens in our scheme, where we avoid that a reaction attack, based on decryption failures, is successful by using ephemeral keys or changing them before the attack can be applied.

1.3.1 Description of the Primitives

The main functions of LEDAkem are described next.

Key generation. Both private and public keys are formed by binary matrices. These matrices, in their turn, are formed by $p \times p$ circulant blocks.

Secret key. The key generation input is formed by:

- The circulant block size p (usually in the order of some thousands bits).
- The integer n_0 (usually between 2 and 4), representing the number of circulant blocks forming the matrix H.
- The integer d_v, representing the row/column weight (usually between 15 and 25) of the circulant blocks forming the matrix H.
- The vector of integers $\bar{m} = [m_0, m_1, \ldots, m_{n_0-1}]$, representing the row/column weights (each entry usually smaller than 10) of the circulant blocks forming the matrix Q.

Given these inputs, the secret key is obtained as follows.

First, n_0 sparse circulant matrices with size p are generated at random. Each of them has row/column weight d_v. We denote such matrices as $H_0, H_1, \ldots, H_{n_0-1}$. The secret low-density parity-check matrix H is then obtained as

$$H = [H_0|H_1|H_2|\ldots|H_{n_0-1}]\,. \tag{1.8}$$

The size of H is $p \times n_0 p$. Other n_0^2 sparse circulant blocks $Q_{i,j}$ are then randomly generated to form the secret sparse matrix

$$Q = \begin{bmatrix} Q_{0,0} & Q_{0,1} & \cdots & Q_{0,n_0-1} \\ Q_{1,0} & Q_{1,1} & \cdots & Q_{1,n_0-1} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{n_0-1,0} & Q_{n_0-1,1} & \cdots & Q_{n_0-1,n_0-1} \end{bmatrix}. \tag{1.9}$$

The row/column weight of each block $Q_{i,j}$ is fixed according to the following matrix:

$$w(Q) = \begin{bmatrix} m_0 & m_1 & \dots & m_{n_0-1} \\ m_{n_0-1} & m_0 & \dots & m_{n_0-2} \\ \vdots & \vdots & \ddots & \vdots \\ m_1 & m_{n_0-1} & \dots & m_0 \end{bmatrix}, \tag{1.10}$$

such that each row and each column of Q has weight $m = \sum_{i=0}^{n_0-1} m_i$.

The choice of the weights $\bar{m} = [m_0, m_1, \cdots, m_{n_0-1}]$ is very important since, as we will prove in the following theorem, it decides whether Q is singular or not. In the following, we denote by $\mathbf{\Pi}\{\cdot\}$ the permanent of a matrix, and with $w(\cdot)$ the weight of a polynomial, i.e., the number of its coefficients that are nonzero.

Theorem 1.3.1 Let $p > 2$ be a prime such that $\text{ord}_p(2) = p - 1$ and Q is an $n_0 \times n_0$ matrix of elements of $\mathbb{F}_2[x]/\langle x^p + 1\rangle$; if $\mathbf{\Pi}\{w(Q)\}$ is odd and $\mathbf{\Pi}\{w(Q)\} < p$, then Q is non singular.

Proof. Since each block Q_{ij} is isomorphic to a polynomial $q_{ij}(x) \in \mathbb{F}_2[x]/\langle x^p+1\rangle$, the determinant of the matrix Q is represented as an element of $\mathbb{F}_2[x]/\langle x^p + 1\rangle$, too. Let us denote by $d(x)$ the polynomial associated to the determinant. If the inverse of $d(x)$ exists, then Q is non singular. According to Section 1.1.3, showing that $d(x)$ has odd weight and $d(x) \neq \Phi(x) = x^{p-1}+x^{p-2}+\cdots+1$ is enough to guarantee that it is invertible. In general, when we are considering two polynomials $a(x)$ and $b(x)$, with $w(a(x)) = w_a$ and $w(b(x)) = w_b$, the following statements hold:

i. $w(a(x)b(x)) = w_a w_b - 2c_1$, where c_1 is the number of cancellations of pairs of monomials with the same exponent resulting from multiplication;

ii. $w(a(x) + b(x)) = w_a + w_b - 2c_2$, where c_2 is the number of cancellations due to monomials with the same exponent appearing in both polynomials.

The determinant $d(x)$ is obtained through multiplications and sums of the elements $q_{ij}(x)$ and, in case of no cancellations, has weight equal to $\mathbf{\Pi}\{w(Q)\}$. If some cancellations occur, considering statements i) and ii) above, we have that $w(d(x)) = \mathbf{\Pi}\{w(Q)\} - 2c$, where c is the overall number of cancellations. So, even when cancellations occur, $d(x)$ has odd weight only if $\mathbf{\Pi}\{w(Q)\}$ is odd. In addition, the condition $\mathbf{\Pi}\{w(Q)\} < p$ guarantees that $d(x) \neq \Phi(x)$, since $w(\Phi(x)) = p$. ■

With this result, we can guarantee that, when the sequence $\bar{m}$ is properly chosen, the matrix Q is always non singular. As we will discuss in the following, a non-singular matrix Q is necessary for key generation to be successful.

Definition 1.3.1 The Secret Key (SK) of LEDAkem is formed by $\{H, Q\}$.

Since both H and Q are formed by sparse circulant blocks, it is convenient to represent each of these blocks through the indexes of the symbols 1 in their first row. Each index of this type requires $\lceil \log_2(p) \rceil$ bits to be stored. If we consider that the circulant blocks in any block row of Q have overall weight $m = \sum_{i=0}^{n_0-1} m_i$, the size of SK in bits is

$$S_{\text{SK}} = n_0 (d_v + m) \lceil \log_2(p) \rceil \tag{1.11}$$

In practice, the secret matrices are generated through a Deterministic Random Bit Generator (DRBG), seeded with a bit string extracted from a True Random Number Generator (TRNG).

In this case, to obtain H and Q it is sufficient to know the TRNG extracted seed of the DRBG that has been used to generate the positions of their non-null coefficients. This approach allows reducing the size of the secret key to the minimum required, as it is assumed that the TRNG output cannot be further compressed. The entity of the reduction depends on the values of the parameters involved in eq. (1.11).

Public key Starting from H and Q, the following binary matrices are computed. First of all, the matrix L is obtained as

$$L = HQ = [L_0|L_1|L_2|\dots|L_{n_0-1}]\,. \tag{1.12}$$

If both d_v and m are odd, then L_{n_0-1} has full-rank. In fact, $L_{n_0-1} = \sum_{i=0}^{n_0-1} H_i Q_{i,n_0-1}$ and has weight equal to $md_v - 2c$ (where c is the number of cancellations occurred in the product). If md_v is odd and $md_v < p$, L_{n_0-1} is non-singular according to Section 1.1.3.

After inverting L_{n_0}, the following matrix is computed:

$$M = L_{n_0-1}^{-1} L = [M_0|M_1|M_2|\dots|M_{n_0-2}|I] = [M_l|I]\,. \tag{1.13}$$

Definition 1.3.2 The Public Key (PK) of LEDAkem is formed by $M_l = [M_0|M_1\ \ |M_2|\dots|M_{n_0-2}]$.

Since the circulant blocks forming M_l are dense, it is convenient to store them through the binary representation of their first row (the other rows are then obtained as cyclic shifts of the first row). The bit-size of the PK hence is

$$S_{\text{PK}} = (n_0 - 1)\, p. \tag{1.14}$$

Encryption The plaintext is intended to be a secret value, k_s, randomly generated by Bob, to be shared with Alice. The encryption inputs are:

- The values of n_0 and p, from which $n = n_0 p$ is computed.
- The number of intentional errors $t \ll n$.

Bob generates, privately, a random binary vector e, with length of $n = n_0 p$ bits and Hamming weight t. Given a Key Derivation Function (KDF), the shared secret key k_s is generated from e as $k_s = \text{KDF}(e)$.

Definition 1.3.3 In order to share the secret e, Bob fetches Alice's PK M_l and computes

$$s = [M_l|I]\, e^T \tag{1.15}$$

where ^{T} denotes transposition. The $p \times 1$ syndrome vector s is then sent to Alice.

Decryption In order to perform decryption, Alice must recover e from s. The latter can be written as

$$s = Me^T = L_{n_0-1}^{-1} L e^T = L_{n_0-1}^{-1} HQe^T. \tag{1.16}$$

The first decryption step for Alice is computing

$$s' = L_{n_0-1} s = HQe^T. \tag{1.17}$$

For this purpose, Alice needs to know L_{n_0-1} that, according to eq. (1.12), is the last circulant block of the matrix HQ. Hence, it can be easily computed from the SK. If we define the *expanded error vector* as

$$e' = eQ^T, \tag{1.18}$$

we have

$$s' = He'^T. \tag{1.19}$$

Hence, QC-LDPC decoding through H can be exploited for recovering e' from s'. QC-LDPC decoders are not bounded distance decoders, and some DFR must be tolerated. However, the system parameters can be chosen such that the DFR is acceptably small. For this purpose, the average decoding radius of the private code must be sufficiently larger than the Hamming weight of e', which is approximately equal to $m\,t$ (due to the sparsity of Q and e). Then, multiplication by $\left(Q^T\right)^{-1}$ would be needed to obtain e from e', that is,

$$e = e' \left(Q^T\right)^{-1}. \tag{1.20}$$

However, by exploiting the efficient decoding algorithms described in Section 1.3.2, this last step can be avoided, which also allows for avoiding to store or compute $\left(Q^T\right)^{-1}$ as part of the secret key. In fact, the decoding algorithm described in Section 1.3.2 allows recovering e directly from eq. (1.17), by performing decoding of s' through H and taking into account the effect of Q. Then, the secret key is recovered as $k_s = \mathrm{KDF}(e)$.

In case a decoding error occurs, the decryption procedure derives the shared secret as $\mathrm{KDF}(s)$, as described in [30]. In this case, Bob will become aware of the decoding failure upon reception of the first message sent by Alice encrypted with the incorrectly derived shared secret.

1.3.2 An efficient decoding algorithm for LEDAkem

While it is possible to perform decoding of the expanded error vector e' employing a classic BF decoder such as the one described in Algorithm 1.1.1, such a choice would not exploit to the utmost the correction power of the QC-LDPC code at hand. Indeed, the error bits in e' are not uniformly distributed; instead their positions depend on the positions of the ones in Q^T, which are known to the decoder.

Starting from classical BF, we have developed an improved decoder that is specifically designed for LEDAkem, which takes into account the fact that the positions of the ones in the expanded error vector e' are influenced by the value of Q^T, as e' is equivalent to a random error vector e with weight t multiplied by Q^T. Since this improved decoder takes into account such a multiplication by the transpose of matrix Q to estimate with greater efficiency the locations of the bits to flip, we denote it as *Q-decoder*.

The inputs of the Q-decoder are the syndrome s' according to eq. (1.17) and the matrices H and Q according to eq. (1.8) and eq. (1.9), respectively. The output of the decoder is a $1 \times n$ vector $\hat{e}$ or a decoding failure, where $\hat{e}$ represents the decoder estimate of the error vector e appearing in eq. (1.17). The decoding procedure goes through a maximum of l_{max} iterations, each iteration is fed with $s^{(l-1)}$ and $\hat{e}^{(l-1)}$, and outputs $s^{(l)}$ and $\hat{e}^{(l)}$. The initialization is performed by setting $s^{(0)} = s'^T$ and $\hat{e}^{(0)} = 0_n$, where 0_n is the length-n vector with all-zero entries. The l-th iteration of the Q-decoder performs the following operations:

i. Compute $\Sigma^{(l)} = [\sigma_1^{(l)}, \sigma_2^{(l)}, \cdots, \sigma_n^{(l)}] = s^{(l-1)}H$ (this multiplication is performed lifting all the elements of both $s^{(l-1)}$ and H in the integer domain $\mathbb{Z}$, hence $\Sigma^{(l)}$ is a vector of integers having entries between 0 and d_v).

ii. Compute $R^{(l)} = [\rho_1^{(l)}, \rho_2^{(l)}, \cdots, \rho_n^{(l)}] = \Sigma^{(l)}Q$ (this multiplication is performed in $\mathbb{Z}$ as well).

iii. Define $b^{(l)} = \max_{j=1,2,\cdots,n}\left\{\rho_j^{(l)}\right\}$ and $\Im^{(l)} = \left\{v \in [1,n] |\ \rho_v^{(l)} = b^{(l)}\right\}$.

iv. Update $\hat{e}^{(l-1)}$ as

$$\hat{e}^{(l)} = \hat{e}^{(l-1)} + \textstyle\sum_{v\in\Im^{(l)}} q_v,$$

where q_v is the v-th row of Q^T.

v. Update the syndrome as

$$s^{(l)} = s + \hat{e}^{(l)}H^T.$$

vi. If the weight of $s^{(l)}$ is zero then stop decoding and return $\hat{e}^{(l)}$.

vii. If $l + 1 \leq l_{max}$ then increment l and go back to step i), otherwise stop decoding and return a decoding failure.

As in the classical BF decoder, the first step of this algorithm computes the vector $\Sigma^{(l)}$. Each entry of this vector counts the number of unsatisfied parity-check equations corresponding to that bit position, and takes values in $[0; d_v]$. This evaluates the likelihood that the entry of e' at the same position is one. Differently from classical BF, in step ii) the correlation $R^{(l)}$ between these likelihoods and the rows of Q^T is computed. In fact, the expanded error vector $e' = eQ^T$ can be written as the sum of the rows of Q^T indexed by the support of e, that is

$$e' = \sum_{j\in\Psi\{e\}} q_j \tag{1.21}$$

where $\Psi\{e\}$ denotes the support of e.

Since both Q and e are sparse (that is, $m, t \ll n$), cancellations between ones are very unlikely. When the correlation between $\Sigma^{(l)}$ and a generic row q_v of Q^T is computed, two cases may occur:

- If $v \notin \Psi\{e\}$, then it is very likely that q_v has a very small number of common ones with all the rows of Q^T forming e', hence the correlation is small.
- If $v \in \Psi\{e\}$, then q_v is one of the rows of Q^T forming e', hence the correlation is large.

The main difference with classical BF is that, while in the latter all error positions are considered as independent, the Q-decoder exploits the correlation among expanded errors which is present in LEDAkem, due to the effect of Q^T. This allows to achieve important reductions in the number of decoding iterations. As a further advantage, this decoder allows recovering e, besides e', without the need of computing and storing the inverse of matrix Q^T.

1.3.3 Choice of the Q-decoder decision thresholds

One important aspect affecting performance of Q-decoders is the choice of the threshold values against which the correlation is compared at each iteration. As described in Section 1.3.2, a natural choice is to set the threshold used at the l-th iteration equal to the element with maximum value of the correlation $R^{(l)}$. This strategy ensures that only those few bits that have maximum likelihood of being affected by errors are flipped during each iteration, thus achieving the lowest DFR. However, it has some drawbacks in terms of complexity, since the computation of the maximum correlation must be performed at each iteration.

Therefore, we consider a different strategy, which allows computing the threshold values on the basis of the syndrome weight at each iteration. According to this approach, during an iteration it is sufficient to compute the syndrome weight and read the corresponding threshold value from a look-up table. This strategy still allows to achieve a sufficiently low DFR, but within a significantly smaller number of decoding iterations.

Let us consider the l-th iteration of the Q-decoder, and denote by t_l the weight of the error vector $e^{(l)}$ and with t'_l the weight of the corresponding expanded error vector $e'^{(l)} = e^{(l)}Q^T$. Let us introduce the following probabilities [5]

$$p_{ci}(t'_l) = \sum_{j\,=\,0,\ j\ \text{odd}}^{\min\left[n_0 d_v - 1, t'_l\right]} \frac{\binom{n_0 d_v - 1}{j}\binom{n - n_0 d_v}{t'_l - j}}{\binom{n-1}{t'_l}} \tag{1.22}$$

$$p_{ic}(t'_l) = \sum_{j\,=\,0,\ j\ \text{even}}^{\min\left[n_0 d_v - 1, t'_l - 1\right]} \frac{\binom{n_0 d_v - 1}{j}\binom{n - n_0 d_v}{t'_l - j - 1}}{\binom{n-1}{t'_l - 1}} \tag{1.23}$$

where:

- $p_{ci}(t'_l)$ is the probability that a codeword bit is error-free and a parity-check equation evaluates it wrongly;
- $p_{ic}(t'_l)$ is the probability that a codeword bit is in error and a parity-check equation evaluates it correctly.

In both cases, the syndrome bit is equal to 1.

The probability that each syndrome bit is equal to 1 can be therefore computed as $p_{ic}(t'_l) + p_{ci}(t'_l)$, so the average syndrome weight at iteration l can be computed as

$$w_s^{(l)} = E\left[wt\left\{s^{(l)}\right\}\right] = \left[p_{ic}(t'_l) + p_{ci}(t'_l)\right] p \tag{1.24}$$

where $wt\{\cdot\}$ denotes the Hamming weight. Since both the parity-check matrix and the error vector are sparse, the probability of $wt\left\{s^{(l)}\right\}$ being significantly different from $w_s^{(l)}$ is negligible.

So, eq. (1.24) allows predicting the average syndrome weight starting from t'_l. In order to predict how t'_l varies during iterations, let us consider the i-th codeword bit and the corresponding correlation value $\rho_i^{(l)}$ at iteration l. The probability that such a codeword bit is affected by an error can be

written as

$$P\left\{e_i = 1|\rho_i^{(l)}\right\} = \frac{P\left\{e_i = 1, \rho_i^{(l)}\right\}}{P\left\{\rho_i^{(l)}\right\}} = \\ = \frac{P\left\{e_i = 1, \rho_i^{(l)}\right\}}{P\left\{e_i = 1, \rho_i^{(l)}\right\} + P\left\{e_i = 0, \rho_i^{(l)}\right\}} = \\ = \left(1 + \frac{P\left\{e_i = 0, \rho_i^{(l)}\right\}}{P\left\{e_i = 1, \rho_i^{(l)}\right\}}\right)^{-1} \tag{1.25}$$

where e_i is the i-th bit of the error vector used during encryption. After some calculations, we obtain

$$P\left\{e_i = 1|\rho_i^{(l)}\right\} = \frac{1}{1 + \frac{n-t_l}{t_l}\left(\frac{p_{ci}(t_l)}{p_{ic}(t_l)}\right)^{\rho_i^{(l)}}\left(\frac{1-p_{ci}(t_l)}{1-p_{ic}(t_l)}\right)^{md_v-\rho_i^{(l)}}} \tag{1.26}$$

where $p_{ci}(t_l)$ and $p_{ic}(t_l)$ are given in eq. (1.22) and eq. (1.23), respectively, with t_l as argument instead of t'_l.

Adding the i-th row of Q^T to the expanded error vector e' is the same as flipping the i-th bit of the error vector e. Hence, we can focus on e and on how its weight t_l changes during decoding iterations. The values of t_l can be estimated as t'_l/m, due to the sparsity, while those of t'_l can be estimated according to (1.24).

The decision to flip the i-th codeword bit is taken when the following condition is fulfilled

$$P\left\{e_i = 1|\rho_i^{(l)}\right\} > (1+\Delta)P\left\{e_i = 0|\rho_i^{(l)}\right\} \tag{1.27}$$

where $\Delta \geq 0$ represents a margin that must be chosen taking into account the DFR and complexity: increasing Δ decreases the DFR but increases the number of decoding iterations. So, a trade-off value of Δ can be found that allows achieving a low DFR while avoiding unnecessary large numbers of iterations.

Since $P\left\{e_i = 0|\rho_i^{(l)}\right\} = 1 - P\left\{e_i = 1|\rho_i^{(l)}\right\}$, eq. (1.27) can be rewritten as

$$P\left\{e_i = 1|\rho_i^{(l)}\right\} > \frac{1+\Delta}{2+\Delta}. \tag{1.28}$$

$P\left\{e_i = 1|\rho_i^{(l)}\right\}$ is an increasing function of $\rho_i^{(l)}$, hence the minimum value of $\rho_i^{(l)}$ such that eq. (1.28) is satisfied can be computed as

$$b^{(l)} = \min\left\{\rho_i^{(l)} \in [0; md_v], \text{ s.t. } P\left\{e_i = 1|\rho_i^{(l)}\right\} > \frac{1+\Delta}{2+\Delta}\right\} \tag{1.29}$$

and used as the decision threshold at iteration l.

Based on the above considerations, the procedure to compute the decision threshold value per each iteration as a function of the syndrome weight can be summarized as follows:

i. The syndrome weights corresponding to $t'_l = 0, m, 2m, \cdots, mt$ (which are all the possible values of t'_l neglecting cancellations) are computed according to eq. (1.24). These values are denoted as $\{w_s(0), w_s(m), \cdots, w_s(mt)\}$.

ii. At iteration l, given the syndrome weight $\bar{w_s}^{(l)}$, the integer $j \in [0, t]$ such that $w_s(jm)$ is as close as possible to $\bar{w_s}^{(l)}$ is computed.

iii. Consider $t_l = j$ and compute $b^{(l)}$ according to eq. (1.29) and eq. (1.26). The value of $b^{(l)}$, so obtained, is used as the decoding threshold for iteration l.

The above procedure can be implemented efficiently by populating a look-up table with the pairs $\{w_j, b_j\}$, sequentially ordered. During an iteration, it is enough to compute $\bar{w_s}^{(l)}$, search the biggest w_j in the look-up table such that $w_j < \bar{w_s}^{(l)}$ and set $b^{(l)} = b_j$.

We have observed that, moving from the bigger values of w_j to the smaller ones, the threshold values computed this way firstly exhibit a decreasing trend, then start to increase. According to numerical simulations, neglecting the final increase is beneficial from the performance standpoint. Therefore, in the look-up table we replace the threshold values after the minimum with a constant value equal to the minimum itself.

Chapter 2

Security Analysis

2.1 Hardness of the underlying problem

The set of computational decision problems for which an efficient solution algorithm can be devised for a non-deterministic Turing Machine (TM) represents a fruitful computational class from which primitives for asymmetric cryptosystems have been designed. Such a computational class, known as the NP (Nondeterministic Polynomial) class, is characterized by problems for which it is efficient (i.e., there is a polynomial-time algorithm) to verify the correctness of a solution on a deterministic TM, while finding a solution to the problem does not have in general an efficient algorithm on a deterministic machine, hence the computational asymmetry required to build a cryptosystem.

When considering a quantum TM, i.e., the abstract computational model for a quantum computer, the class of problems which can be solved in polynomial time, with the quantum TM providing the correct answer with probability $> \frac{2}{3}$, is known as the Bounded-error Quantum Polynomial time class, BQP [9]. In 1997 Peter Shor proved that the integer factoring problem, which has its decisional version in NP, is effectively in BQP [36], in turn demonstrating that a widely adopted cryptographic trapdoor function can be broken in polynomial time by a quantum computer. Consequentially, to devise a proper post-quantum asymmetric primitive it is crucial to choose a computational problem which resides outside BQP as its underlying foundation. While there is no current formal proof, a sub-class of NP, the NP-complete problem class, is widely believed to contain computational problems not belonging to BQP, thus allowing only a polynomial speedup in their solution with a quantum TM.

LEDAkem is constructed starting from the computational problem of performing the decoding of a syndrome, i.e., deriving the corresponding error vector with a bounded weight for a general linear code, which was shown to be NP-complete in [7]. Indeed, in [7] the authors show that there is no exponentially faster way to compute the error vector of a general linear code than through enumerating and testing all the possible ones, unless P=NP. While the public matrix M_l of the LEDAkem cryptosystem has quasi-cyclic structure, we note that no algorithms currently exist exploiting this regularity to gain an exponential advantage in decoding the corresponding code.

With this statement standing, the security analysis of LEDAkem examines and quantifies the effectiveness of the best known attacks detailing the efficiency of algorithms running on both classical and quantum computers providing non-exponential speedups over an enumerative search for the correct error vector. We remark that currently no algorithm running on either a classical TM or

a quantum TM provides an exponential speedup in solving the computational problem underlying LEDAkem compared to an exhaustive search approach.

2.2 Analysis of the algorithm with respect to known attacks

Reaction Attacks. One of the most effective attacks against the QC-LDPC code-based cryptosystems already established in the literature [4, 5], from which LEDAkem is derived, are the so-called *reaction attacks* proposed in [14]. Reaction attacks exploit the correlation between the DFR of a given code associated to a keypair and the structure of the secret key. To this end, reaction attacks need to estimate the DFR on a set of properly crafted erroneous codewords. LEDAkem prevents reaction attacks altogether using ephemeral keys, since an attacker may only obtain the result of a single decode action with a given keypair. We note that, given the low DFR of the code instances which are proposed for LEDAkem even the occasional reuse of a key will not allow reaction attacks to threaten the system.

Squaring Attacks. Another potential attack to systems based on QC-LDPC codes is the one presented in [35]. This attack uses a so-called *squaring technique* to find a a low-weight error vector and thus low-weight codewords more efficiently than with a general Information Set Decoding (ISD) algorithm. This attack, however, is applicable if and only if the size of the circulant blocks p is even. In LEDAkem p is chosen as a prime both as a conservative choice against cryptanalysis exploiting factorization of p, and gaining in terms of efficiency due to the invertibility test reported in Theorem 1.3.1.

Decoding Attacks (DA) and Key Recovery Attacks (KRA). Concerning attacks which aim at solving efficiently the decoding of a message exploiting the public code representation, a prime position is occupied by the ISD approach. The ISD approach attempts at performing the decoding of a general linear code (polynomially) more efficiently than an exhaustive search approach, and was pioneered by Prange in [32]. Subsequent improvements of Prange's algorithm were presented by Lee-Brickell [21], Leon [22] and Stern [37] improving, although polynomially on Prange's original algorithm. Among these variants, the most noteworthy one is the one by Stern [37] which is currently the one best exploiting the speedups provided by quantum computers according to [12]. In particular, a significant portion of Stern's algorithm can be solved employing Grover's algorithm [17] to cut down the running time to the square root of the one needed for a computation on a classical platform. By contrast,when considering efficient execution on classical computers the most efficient ISD turns out to be the Becker-Joux-May-Meurer (BJMM) algorithm proposed in [6] which is part of a family of recent results [8, 25, 28, 31]. As a consequence, the security levels against attackers performing a Decoding Attack (DA) with classical computers have been estimated by considering the work factor of the BJMM algorithm, while the security levels against quantum computer-equipped attackers were computed taking into account Stern's algorithm.

A different approach at attacking the system is the one of efficiently finding low-weight codewords in the dual of the public code of the cryptosystem. It is possible to show that the low weight codeword finding problem is equivalent to the general linear code decoding problem, thus allowing ISD to be retrofit to this task too. In the case of LEDAkem we note that the matrix $L = HQ$, which is generally sparse, is a valid parity-check matrix for the public code. Since the rows of L are sparse codewords of the code generated by M (that is, the dual code of the public code), and have

weight in the order of $n_0 d_v m$, an attacker may search for them in the dual of the public code. An opponent may thus exploit an efficient algorithm for the search of low-weight codewords in linear block codes, thus performing a Key Recovery Attack (KRA) on L, row by row. Then, because of the sparsity of L, the opponent may succeed in separating H from Q and recovering the secret key. Alternatively, the attacker may just attempt to perform the decoding action employing the entire L, which has low weight, as a parity-check matrix.

We defend LEDAkem from both DAs and KRAs employing parameters which prevent the low-weight codeword finding from succeeding given a computational power bounded by the desired security level. To this end, we take into account the fact that the nature of the QC codes employed in LEDAkem provides a speedup by a factor $\sqrt{p}$ with respect to the running time of the ISD algorithm employed to perform a general linear code decoding [34]. Instead, when the ISD algorithm is employed with the purpose of retrieving low-weight codewords in L the speedup factor provided by the QC structure of L is p with respect to its application to a general code. Both these speedup factors are taken into account in our estimates of the security level for a given parameter set.

Quantum Stern's algorithm. Considering the fact that Stern's algorithm [37] is the one best suited for quantum computer execution, and will thus be employed to determine the parameters of LEDAkem, we briefly resume the results in [12], describing how the application of Grover's algorithm to ISD can be taken into account when computing the complexity of KRAs and DAs.

ISD is an algorithm $\mathcal{A}(\mathcal{C}(n,k),w)$ taking as input a code $\mathcal{C}(n,k)$ with length n, dimension k, and tries to find a codeword of weight w or, equivalently, an error vector with weight w given the code and the corresponding syndrome.

In LEDAkem, employing ISD to perform a general decoding will have it acting on an $n_0 p$ bits long code, with dimension $(n_0-1)p$, trying to correct t errors, while employing it to perform low-weight codeword finding is equivalent to running it on a code $n_0 p$ bits long, with dimension p, trying to correct $n_0 d_v m$ errors.

The basic structure of each ISD algorithm is essentially the same, and is based on the identification of an *information set*, that is, a set of k linearly independent columns of the generator matrix of the code. Recovering the entries of the error vector affecting this set is enough to reconstruct the whole error vector. The algorithm must be run iteratively, and each iteration has a probability of success $p_{\mathcal{A}}$. Thus, the expected number of iterations that makes the attack successful is $\frac{1}{p_{\mathcal{A}}}$. The probability $p_{\mathcal{A}}$ is obtained as the product of p_{inv} and p_e, where p_{inv} is the probability that an iteration of ISD has selected a set of k linearly independent vectors, while p_e is the probability that the error vector entries affecting the selected set can be recovered. It can be proven that p_{inv} converges to $p_{inv} \approx 0.29$ as the size of the binary matrix being inverted increases, while for p_e we have

$$p_e = \frac{\binom{w}{2m}\binom{n-w}{k-2m}\binom{2m}{m}\binom{n-k-w+2m}{l}}{4^m\binom{n}{k}\binom{n-k}{l}}$$

according to [37], where l and m are parameters which influence the complexity of the algorithm and must be optimized to minimize the value of p_e.

Taking into account the speedup following from the application of Grover's algorithm to Stern's algorithm, it follows that the algorithm is successful after performing only $\frac{\pi}{4}\sqrt{\frac{1}{p_{\mathcal{A}}}} = \frac{\pi}{4}\sqrt{\frac{1}{p_{inv}p_e}}$ iterations on average, instead of $\frac{1}{p_{inv}p_e}$. Let us define:

- c_{dec} as the cost in qubit operations of decoding the input qubits to the inputs of the classical algorithm which must be performed whenever an iteration completed on the quantum computer;
- c_{it} as the number of bit operations needed to perform an iteration of the classical Stern's algorithm;
- c_{inv} as the cost of inverting the matrix obtained with the k columns selected during the iteration; in fact, because a quantum implementation of Stern's algorithm must be performed entirely with revertible operations, skipping an iteration is not possible, even if the selected k-columns do not correspond to an information set (i.e., they are not linearly independent).

By taking the conservative assumption that a qubit operation has the same cost of a bit operation, it is possible to express the amount of operations required to execute Stern's algorithm on a quantum computer as

$$\frac{\pi}{4}\sqrt{\frac{1}{p_{inv}p_e}}(c_{dec}+c_{inv}+c_{it}) \tag{2.1}$$

Estimating the actual value of c_{dec} can be very hard, since it depends on the size of the input given to $\mathcal{A}$. For example, some input parameters can be fixed (in this case, the number of bits needed to represent the input given to $\mathcal{A}$ decreases) but, at the same time, the value of p_e might get lower (since, in this case, we might not consider an optimal input choice). While estimates for c_{dec} have put it in the 2^{30} range [12], we conservatively consider $c_{dec}=0$. Finally, to compute the two remaining computational costs, we refer to the following expressions from [37]:

$$c_{it} = 2lm\binom{k/2}{m} + 2m(n-k)\binom{k/2}{m}^2 2^{-l} \tag{2.2}$$

$$c_{inv} = \frac{1}{2}(n-k)^3 + k(n-k)^2 \tag{2.3}$$

BJMM algorithm complexity. As already mentioned, when only classical computers are available, the most efficient ISD algorithm turns out to be the BJMM algorithm proposed in [6]. A precise estimation of the work factor of this algorithm in the finite-length regime can be found in [19], and it has been used to compute the work factor of attacks based on ISD against the proposed instances of LEDAkem, when performed with classical computers. While the complete expression of the computational complexity of the BJMM algorithm is rather complex, we point out that a simple expression providing an approximate but fairly intuitive expression for it is reported in [11]: 2^{cw}, where $c=\log_2\frac{1}{1-\frac{k}{n}}$.

2.3 System parameters for the required security categories

Nine sets of parameters for LEDAkem are proposed, clustered into in three classes corresponding to different security guarantees. Each one of the three instances in each class correspond to a different value of n_0 $(2,3,4)$, yielding a different balance between performance and public key size. The parameters of the nine instances of LEDAkem are reported in Table 2.1 for the NIST required security categories 1, 3 and 5, respectively. We assume that the security requirements of category

2 can be fulfilled employing category 3 parameters, and the security requirements for category 4 are fulfilled by category 5 parameters. In the table, the superscript (pq) denotes that the attack work factor has been computed taking into account quantum speedups due to Grover's algorithm, while the superscript (cl) denotes that only classical computers have been considered in evaluating the attack work factor.

For each security category and considered value of n_0, we computed the minimum values of d_v md_v and t which ensure the desired the security level against KRAs and DAs, respectively. Then we selected a value for the circulant block size p as a prime with $\texttt{ord}_2(p) = p - 1$ to provide efficient invertibility tests for circulant blocks. We have checked whether tm errors can be corrected by the private code through Q-decoding maintaining a sufficiently low DFR via Montecarlo simulations. In particular, we targeted 10^{-8} as an acceptable DFR. Otherwise, we have increased the value of p and repeated the procedure.

In order to obtain a preliminary estimate of the value of p to speed up the design procedure, we exploited the BF asymptotic thresholds supplied in [5]. These thresholds allow, given the code size n, dimension k and density d_v, to compute an estimate of the biggest weight of an error vector which can be corrected by the code, using a classical BF decoder.

This approach can be used also for predicting the correcting capability of LEDAkem: indeed, we have observed that, in the waterfall region of the Q-decoder, the DFR of the system, when weight-t error vectors are used, can be approximated by the one of a BF-decoder, taking as input a parity check matrix in the same form as eq. (1.8) and having circulant blocks with weight equal to md_v. Thus, the BF threshold can be used to predict the correcting capability of a LEDAkem instance: if the theoretical threshold is below the value of t, then a bigger value of p must be chosen. We want to stress that the described DFR estimation procedure is only approximated, and Montecarlo simulations must be performed in order to evaluate the actual DFR of the system.

In order to make a conservative design choices for the parameters, we have considered some margin in the complexity estimates of the attacks, such that the actual security level for these instances is larger than the target one. This also accounts for possible (though rare) cancellations occurring in L when computed as the product of H and Q, which may yield a row weight slightly smaller than md_vn_0, influencing the resistance to KRAs. The values of d_v have been chosen greater than 15 in order to avoid codes having small minimum distances.

To ensure that both H and Q have maximum rank, we chose d_v odd and $[m_0, m_1, \cdots, m_{n_0-1}]$ have been chosen such that the value of $\mathbf{\Pi}\{w(Q)\}$ is odd and smaller than p, allowing theorem 1.3.1 to hold. Indeed since, $L = HQ$ is a valid parity-check matrix for the public code, a singular Q, may result in the rank of L being lower than p, leading to a code with a co-dimension lower than p. When multiple choices of m and d_v were possible to achieve the same security level, we have selected those with the lowest values of the product md_v, as this is known to enhance the error correcting capability of the private code.

The system parameters design procedure can thus be summarized as follows:

i. pick a desired security level SL expressed as the $\log_2$ of the number of operations to be performed and a number of circulant blocks n_0;

ii. consider the computational effort of performing a decoding via ISD and compute the minimum number of errors $\hat{t}$, in order to ensure that DAs take more than 2^{SL} operations;

iii. consider the computational effort of performing a KRA via ISD and compute the minimum

Table 2.1: Parameters for LEDAkem and estimated computational efforts to break a given instance as a function of the security category and number of circulant blocks n_0

Category	$\mathbf{n_0}$	p	$\mathbf{d_v}$	$[\mathbf{m_0}, \cdots, \mathbf{m_{n_0-1}}]$	t	$\mathbf{SL_{DA}^{(pq)}}$	$\mathbf{SL_{KRA}^{(pq)}}$	$\mathbf{SL_{DA}^{(cl)}}$	$\mathbf{SL_{KRA}^{(cl)}}$	DFR
1	2	27,779	17	[4, 3]	224	135.43	134.84	217.45	223.66	$\approx 8.3 \cdot 10^{-9}$
	3	18,701	19	[3, 2, 2]	141	135.63	133.06	216.42	219.84	$\lesssim 10^{-9}$
	4	17,027	21	[4, 1, 1, 1]	112	136.11	139.29	216.86	230.61	$\lesssim 10^{-9}$
2–3	2	57,557	17	[6, 5]	349	200.47	204.84	341.52	358.16	$\lesssim 10^{-8}$
	3	41,507	19	[3, 4, 4]	220	200.44	200.95	341.61	351.57	$\lesssim 10^{-8}$
	4	35,027	17	[4, 3, 3, 3]	175	200.41	201.40	343.36	351.96	$\lesssim 10^{-8}$
4–5	2	99,053	19	[7, 6]	474	265.38	267.00	467.24	478.67	$\lesssim 10^{-8}$
	3	72,019	19	[7, 4, 4]	301	265.70	270.18	471.67	484.48	$\lesssim 10^{-8}$
	4	60,509	23	[4, 3, 3, 3]	239	265.48	268.03	473.38	480.73	$\lesssim 10^{-8}$

value of md_v, in order to ensure that KRAs take more than 2^{SL} operations; denote this value as $\hat{d}'_v$;

iv. compute an initial value of p such that $\mathtt{ord}_p(2) = p-1$ and that the resulting code is expected to be correcting at least t errors via Q-decoder according to asymptotic estimates;

v. choose d_v as an odd number and a set of integers $[m_0, m_1, \cdots, m_{n_0-1}]$ such that $\mathbf{\Pi}\{w(Q)\}$ is odd and smaller than p and $md_v \geq \hat{d}'_v$. If more than a solution is possible pick the one with md_v closest to $\hat{d}'_v$;

vi. simulate the DFR of the code; if it is not sufficiently low, restart from (iv) and choose a larger value of p.

In the steps (ii) and (iii) a temporary value of p is used to compute the work factor of the attacks, which however do not exhibit a significant dependence on p (and so, on n). Nevertheless, at the end of the design process, it is necessary to verify that the final value of p actually yields a work factor of both DAs and KRAs above the target security level.

Table 2.1 reports the values of the parameters derived with the aforementioned procedure for the nine instances of LEDAkem. The DFR were estimated through Montecarlo simulations: for all the parameter sets belonging to categories 3 and 5 we computed 10^8 decoding actions without encountering a single decoding error. For the parameter set corresponding to category 1, with $n_0 = 2$ circulant blocks we obtained an estimate of the DFR of $\approx 8.3 \cdot 10^{-9}$, having encountered 20 decoding errors over $2.394 \cdot 10^9$ decoding actions. For the parameters corresponding to category 1 and $n_0 \in \{3, 4\}$ we encountered no decoding errors during the computation of 10^9 decoding actions.

2.4 Properties of the cryptosystem

From the standpoint of formal security guarantees, LEDAkem enjoys the property that its ciphertext is indistinguishable from a random string under a chosen plaintext attack model, i.e., LEDAkem provides a KEM with Indistinguishability under Chosen Plaintext Attack (IND-CPA), under the random oracle model, with a polynomially computationally bound adversary. Indeed, assuming that the KDF employed to derive the shared secret from the encrypted error vector can be modeled as a random oracle, the attacker, supplied with a ciphertext and an alleged shared secret

value, is not able to understand whether or not the ciphertext is actually derived from the alleged shared secret or not unless he is able to perform a further call to an encryption oracle. Furthermore, we note that the LEDAkem primitive is amenable to be employed to build an Indistinguishability under Chosen Ciphertext Attack (IND-CCA) KEM+DEM scheme, as described in [30].

Furthermore, we note that LEDAkem prevents the trivial ciphertexts malleability due to the linearity property of error correcting codes. Indeed, despite an attacker may be able to alter the ciphertext so that it decrypts to a valid error vector e, the shared secret is derived via a hash based KDF, which prevents him from choosing the actual value of the shared secret, as it would imply that the attacker is able, given an output of the KDF, to provide a valid pre-image for it.

Given the running times of the key generation algorithm, LEDAkem can be employed to provide Perfect Forward Secrecy (PFS) as described in [20]. Indeed, since LEDAkem is proposed as an ephemeral-key KEM, it is sufficient to erase the keypairs as soon as the session key has been established, and to erase the session key at the end of the communication to achieve PFS.

While LEDAkem does not provide authentication features per se, it is possible to exploit general constructions such as the one proposed in [15] to turn an IND-CCA KEM into an Authenticated Key Exchange (AKE).

Finally, in case the goodness of the sender TRNG is questionable, the LEDAkem protocol can be repeated in both directions to exchange two partial secret keys, k_{s_1} and k_{s_2}, each one generated by an endpoint. Then, the session key is obtained as $\mathrm{KDF}(k_{s_1} \mid k_{s_2})$, thus compensating for possible TRNG shortcomings of one of the involved parties.

Risks in case of keypair reuse

While LEDAkem uses ephemeral keys that are meant for single use, it is possible that implementation accidents lead to a reuse of the same keypair more than once. The main threat in case of keypair reuse is the reaction attack described in [14]: the attack relies on the correlation existing between the structure of the parity matrix of QC-LDPC code and its DFR which raises the DFR above the average whenever the error pattern matches the structure of the parity matrix in a significant amount. However, for the attack to succeed, the attacker needs to reliably estimate the decoding failure rate for a large set of carefully crafted or selected error vectors, and this can be done only after a minimum (and large) number of decoding errors is observed. Considering that, in order to wait for a single decoding error, the attacker must wait (on average) for DFR^{-1} cyphertexts, occasional keypair reuses do not represent a problem, given that the DFR of the codes in the proposed instances of LEDAkem are in the 10^{-8} to 10^{-9} range. This in turn allows a very graceful degradation in case of an accidental keypair reuse.

Resistance to multi-key attacks

No multi-key attacks have been reported against these systems and, more in general, against McEliece/Niederreiter cryptosystems.

Chapter 3

Implementation strategies and performance analysis

3.1 Procedural description of the LEDAkem primitives

To the end of providing an efficient implementation of LEDAkem, we represented each circulant block as a polynomial in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$ thanks to the isomorphism previously described.
Consequently, all the involved block circulant matrices are represented as matrices of polynomials in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$. The polynomials are materialized employing a bit-packed form of their binary coefficients in all the cases where the number of non null coefficients is high. In case a polynomial has a low number of non null coefficients with respect to the maximum possible, i.e., its corresponding circulant matrix is sparse, we represent it materializing only the positions of the one coefficients as integers.

The LEDAkem key generation algorithm is reported in Algorithm 3.1.1, which takes as input the QC-LDPC code parameters and yields a private- and public-key pair. The first operation performed by the algorithm is the extraction of a private key SK as a random value, `rndPrivateMatricesSeed` generated from a TRNG (line 2 in Algorithm 3.1.1) and long enough to provide the desired security margin when deriving the secret matrices H and Q. The approach adopted to generate both the $1 \times n_0$ block matrix H and the $n_0 \times n_0$ block matrix Q is to expand the value `rndPrivateMatricesSeed` employing the NIST provided seed expander built on AES-256-CTR to draw random position for the asserted coefficients of the polynomials (blocks) of the matrices (line 2 in Algorithm 3.1.1). In case a duplicate position is drawn, it is discarded and a fresh position is drawn anew. The weights of the blocks of Q are designed in such a fashion that it is always invertible (see Section 1.3.1). We evaluated that repeating this generation process does not have a significant impact on the decryption phase, and thus opted to store only the value of `rndPrivateMatricesSeed` as the cipher private key SK (line 11 in Algorithm 3.1.1). Indeed, the size of H and Q during the computation of the decryption algorithm is still rather small, as their sparsity allows for a compact representation in memory, where only the position of the one-valued coefficients are materialized. Given the bit-sizes of the parameters of the cryptosystem, the positions can be materialized as either 16-bit or 32-bit integers depending on the chosen values for n_0 and p. The next step of the key generation algorithm is to compute the $1 \times n_0$ block matrix $L = HQ = [L_0, L_1, \ldots, L_{n_0-1}]$ (lines 3–6 in Algorithm 3.1.1). We recall that, given the choice of d_v odd, and $\sum_{i=0}^{n_0-1} m_i$ odd, the invertibility of the last block of L, L_{n_0-1} is guaranteed a priori (see Section 1.3.1). Therefore,

Algorithm 3.1.1: LEDAkem Keygen

Input: p, n_0, n, k: QC-LDPC code parameters, where p denotes a circulant block size (in bit), and n_0 denotes the number of circulant blocks of the $1 \times n_0$ parity-check matrix of the code. $n = pn_0$ (bit) denotes the codeword size, while $k = p(n_0 - 1)$ (bit) denotes the information word size.
d_v: odd weight of each circulant block of the parity-check matrix $H = [H_0 \mid H_1 \mid \ldots \mid H_{n_0-1}]$ to be generated
$[m_0, \ldots, m_{n_0-1}]$: weight of each block of the first row of the $n_0 \times n_0$ circulant block matrix $Q = \begin{bmatrix} Q_{0,0} & \cdots & Q_{0,n_0-1} \\ \vdots & \ddots & \vdots \\ Q_{n_0-1,0} & \cdots & Q_{n_0-1,n_0-1} \end{bmatrix}$ to be generated – with $\left(\sum_{i=0}^{n_0-1} m_i\right)$ odd

Output: (SK, PK) generated private-key/public-key pair

1 `rndPrivateMatricesSeed` $\leftarrow$ TRNG()
2 $H, Q \leftarrow$ GenerateHQ$(n_0, d_v, [m_0, \ldots, m_{n_0-1}],$ `rndPrivateMatricesSeed`$)$
3 **for** $i = 0$ **to** $n_0 - 1$ **do**
4 $\quad L_i \leftarrow 0$ `// null polynomial in` $\mathbb{F}_2[x]/\langle x^p + 1\rangle$
5 $\quad$ **for** $j = 0$ **to** $n_0 - 1$ **do**
6 $\quad\quad L_i \leftarrow L_i + H_j Q_{j,i}$ `// polynomial mul. and add. in` $\mathbb{F}_2[x]/\langle x^p + 1\rangle$
7 `LInv` $\leftarrow$ ComputeInverse(L_{n_0-1}) `// multiplicative inverse in` $\mathbb{F}_2[x]/\langle x^p + 1\rangle$
8 **for** $i = 0$ **to** $n_0 - 2$ **do**
9 $\quad M_i \leftarrow$ `LInv` L_i `// polynomial multiplication in` $\mathbb{F}_2[x]/\langle x^p + 1\rangle$
10 PK $\leftarrow [M_0 \mid \ldots \mid M_{n_0-2}]$
11 SK $\leftarrow$ `rndPrivateMatricesSeed`
12 **return** (SK, PK)

the computation of `LInv` $= L_{n_0-1}^{-1}$ is performed with a single call to the polynomial inversion algorithm (line 7 in Algorithm 3.1.1). Finally, the public key PK is generated as a $1 \times n_0 - 1$ block matrix, $[M_0 \mid \ldots \mid M_{n_0-2}]$, through multiplying `LInv` by all-but-the-last blocks of L (lines 8–9 in Algorithm 3.1.1). The last multiplication can be avoided as it will yield the identity matrix which is thus not stored in the PK (line 10 in Algorithm 3.1.1).

The encryption process for LEDAkem is remarkably simple, and is reported in Algorithm 3.1.2. Starting from a suitably sized random value, `rndErrorSeed`, extracted from a TRNG (line 1 in Algorithm 3.1.2), the encryption procedure generates an $n_0 p$ bit random error vector with weight t, obtaining the positions of the t asserted bits expanding the `rndErrorSeed` value via the NIST provided AES-based seed expander. The generation of the positions of the asserted bits keeps into account the meaning given to the mentioned random binary vector of a $1 \times n_0$ block error vector, $e = [e_0, e_1, \ldots, e_{n_0-1}]$, where each component, $e_i, 0 \leq i < n_0$, is meant to be an element in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$ (line 2 in Algorithm 3.1.2). Once the error vector e is generated, it is multiplied by the $1 \times n_0$ block matrix $M = [M_0 \mid M_1 \mid \ldots \mid M_{n_0-2} \mid I]$ (derived from the public key PK) to obtain the syndrome s as the encapsulated (i.e., encrypted) secret error vector (lines $3 - 6$ in Algorithm 3.1.2). Since M is not entirely materialized (indeed, its last block is a $p \times p$ identity matrix – i.e., the constant non null polynomial – and it is not stored), the multiplication $[M_0 \mid \ldots \mid M_{n_0-2} \mid I]e^T$ is done multiplying the first $(n_0 - 1)$ blocks of e with the corresponding blocks of $[M_0 \mid M_1 \mid \ldots \mid M_{n_0-2}]$ and adding the last block of e (i.e., e_{n_0-1}) to the result.
Finally, the shared secret k_s is derived processing the secret error vector e with a KDF having an output of the desired size (line 7 in Algorithm 3.1.2). We chose to employ SHA-3 as our KDF, as

Algorithm 3.1.2: LEDAkem Encrypt

Input: p, n_0, n, k: QC-LDPC code parameters, where p denotes a circulant block size (in bit), and n_0 denotes the number of circulant blocks of the $1 \times n_0$ parity-check matrix of the code. $n = pn_0$ (bit) denotes the codeword size, while $k = p(n_0 - 1)$ (bit) denotes the information word size.
PK $= [M_0 \mid \ldots \mid M_{n_0-2}]$: the public key constituted by the first $n_0 - 1$ blocks of the $1 \times n_0$ block matrix $M = [M_0 \mid \ldots \mid M_{n_0-2} \mid I]$
t: the weight of the error vector decided according to the security level

Output: k_s: the secret shared among the parties
s: syndrome, corresponding to the encapsulated (encrypted) secret error vector

1 `rndErrorSeed` $\leftarrow$ TRNG()
2 $e \leftarrow$ GenerateSecretError(t, `rndErrorSeed`) // $e = [e_0 \mid e_1 \mid \ldots \mid e_{n_0-1}]$
3 $tmp \leftarrow 0$
4 **for** $i = 0$ **to** $n_0 - 2$ **do**
5 $\quad tmp \leftarrow tmp + e_i M_i$
6 $c = tmp + e_{n_0-1}$
7 $k_s \leftarrow \mathrm{KDF}(e)$
8 **return** $\langle k_s, c \rangle$

it provides digest sizes which are easily matched with the required security categories by NIST.

The decryption procedure for LEDAkem, detailed in Algorithm 3.1.3, starts by providing the private key value SK to the NIST approved seed expander, and by recomputing the positions of the asserted coefficients in sparse representations of the secret matrices H and Q, following the same approach employed in the LEDAkem key generation (line 1 in Algorithm 3.1.3). During decryption process only the the computation of the last block of the $1 \times n_0$ matrix $L = HQ = [L_0, \ldots, L_{n_0-1}]$ is required (lines $2-4$ in Algorithm 3.1.3)) Indeed, the (re)derivation of H, Q and L_{n_0-1} is performed with only a minimal ($< 1\%$) computational overhead compared with the computational cost of the decryption operations. Following the derivation of L_{n_0-1} the received encrypted message s, i.e., the syndrome of the QC-LDPC code, is multiplied by L_{n_0-1} to obtain a value $s' = s\, L_{n_0-1}$, which is in turn passed to the Q-decoder procedure, along with the sparse representations of H^T and Q^T (line 6 in Algorithm 3.1.3). If the decoding procedure does not fail in recovering the error vector e, the LEDAkem decryption procedure derives the shared secret k_s processing the error vector with the same KDF employed by the encryption primitive (line 8 in Algorithm 3.1.3). If the decoding procedure fails, the LEDAkem decryption procedure returns a value derived from processing through the same KDF the received encrypted message (line 10 in Algorithm 3.1.3), to the purpose of providing the desired properties to prove the KEM-IND-CCA as reported in [30].

The decoding procedure employed in LEDAkem is detailed in Algorithm 3.1.4 and attempts at reconstructing the secret error vector e from the received syndrome s. To this end, the peculiarity of the Qdecode algorithm is that it directly reconstructs the value of e, where a common bit-flipping decoder would retrieve $e' = eQ^T$, thus requiring a further matrix multiplication to complete the decryption action.

To perform the required syndrome decoding, Qdecode starts by computing the number of unsatisfied parity checks in the current syndrome in the same way a standard bit flipping algorithm does (lines $5-9$ in Algorithm 3.1.4). Our approach to implement this computation is to exploit a sparse

Algorithm 3.1.3: LEDAkem Decrypt

Input: p, n_0, n, k: QC-LDPC code parameters, where p denotes a circulant block size (in bit), and n_0 denotes the number of circulant blocks of the $1 \times n_0$ parity-check matrix of the code. $n = pn_0$ (bit) denotes the codeword size, while $k = p(n_0 - 1)$ (bit) denotes the information word size.
d_v: odd weight of each circulant block of the parity-check matrix $H = [H_0 \mid H_1 \mid \ldots \mid H_{n_0-1}]$ to be generated
$[m_0, \ldots, m_{n_0-1}]$: weight of each block of the first row of the $n_0 \times n_0$ circulant block matrix $Q = \begin{bmatrix} Q_{0,0} & \cdots & Q_{0,n_0-1} \\ \vdots & \ddots & \vdots \\ Q_{n_0-1,0} & \cdots & Q_{n_0-1,n_0-1} \end{bmatrix}$ to be generated – with $\left(\sum_{i=0}^{n_0-1} m_i \right)$ odd
SK: private key
s: received syndrome, corresponding to the encrypted secret error vector

Output: k_s: the secret shared among the parties

1 $H, Q \leftarrow$ GenerateHQ$(n_0, d_v, [m_0, \ldots, m_{n_0-1}], \text{SK})$
2 $L_{n_0-1} \leftarrow 0$
3 **for** $i = 0$ **to** $n_0 - 1$ **do**
4 $\quad L_{n_0-1} \leftarrow L_{n_0-1} + H_i Q_{i,n_0-1}$
5 $s' \leftarrow L_{n_0-1}\, s$
6 e, `decodeOk` $\leftarrow$ QDecode(s', H^T, Q^T)
7 **if** `decodeOk` $=$ *true* **then**
8 $\quad k_s \leftarrow \text{KDF}(e)$
9 **else**
10 $\quad k_s \leftarrow \text{KDF}(s)$
11 **return** k_s

representation for the transposition of the parity check matrix H^T, which is taken as an input and denoted as `Htr` in the algorithm. This, in turn, allows to reduce the number of iterations of the innermost loop of the parity check computation (lines 7–9 in Algorithm 3.1.4) from $\frac{n_0 p}{\texttt{machine\_word}}$ to d_v. For example, considering the case of $n_0 = 2, p = 25931$ on the NIST reference platform (`machine_word` $= 64$) the number of iterations drops from 811 to 17.

The differentiating point between the classical bit flipping algorithm and the Q-decoding concerns how the bits of the codeword being decoded are selected for flipping. Indeed, while employing a classical bit flipping algorithm would flip all the positions in e' having the highest number of unsatisfied parity checks, the Q-decoder exploits the knowledge of the secret matrix Q^T to estimate if a bit flip should be performed. To this end, the Q-decoding computes, for each bit of e being decoded (lines 12–13 in Algorithm 3.1.4) a measure of similarity between the patterns of ones of a row of Q^T, blockwise cyclically shifted by the position of the bit of e itself, and the unsatisfied parity checks vector. If the similarity metric (lines 14–16 in Algorithm 3.1.4) is above a given threshold, both the error vector e and the value of the syndrome s for the next iteration `iter` are updated (lines 18–23 in Algorithm 3.1.4). The value of the aforementioned threshold can be obtained as a piecewise constant function of the current syndrome weight and the code parameters. For efficiency reasons, the function is precomputed and stored as a lookup table (`LutS`) containing pairs (weight, threshold). The Q-decoder computes the weight of the syndrome and determines the highest weight $\bar{\texttt{w}}$ among the ones in the lookup table, which does not exceed the one of the

Algorithm 3.1.4: Qdecode

Input: s': QC-LDPC syndrome, binary vector of size p
`Htr`: transposed parity-check matrix, represented as an $n_0 \times d_v$ integer matrix containing the positions in $\{0, 1, \ldots, p-1\}$ of the set coefficients in the n_0 blocks of $H^T = [H_0^T \mid H_1^T \mid \ldots \mid H_{n_0-1}^T]$
`Qtr`: private matrix, represented as an $n_0 \times m$, $m = \sum_{i=0}^{n_0-1} m_i$ integer matrix containing the positions in $\{0, \ldots, n_0 p - 1\}$ of the asserted coefficients in Q^T rows

Output: e: the decoded error vector with size $n_0 p$
`decodeOk`: Boolean value denoting the successful outcome of the decoding action

Data: `imax`: the maximum number of allowed iterations before reporting a decoding failure
`LutS`: piecewise constant function yielding the value of the bit flipping threshold of similarity, given the syndrome weight.
It is represented as an array of (weight, threshold) pairs for all the boundary values of the piecewise function.

```
1  iter ← 0
2  repeat
3      unsat_pc ← [0 | ... | 0]    // array of n0p counters of unsatisfied parity checks
4      currSynd ← s'
5      for i = 0 to n0 − 1 do
6          for exp = 0 to p − 1 do
7              for h = 0 to dv − 1 do
8                  if GETBLOCKCOEFFICIENT(currSynd, (exp + Htr[i][h]) mod p) = 1 then
9                      unsat_pc[i · p + exp] ← unsat_pc[i · p + exp] + 1
10     w̄ ← MAX({w | (w, th) ∈ LutS ∧ w < WEIGHT(currSynd)})
11     th̄ ← th | (w̄, th) ∈ LutS
12     for i = 0 to n0 − 1 do
13         for exp = 0 to p − 1 do
14             similarity ← 0
15             for k = 0 to m − 1 do
                   // qrow contains the positions of the ones of a row of Q rotated intra-block by j
16                 qrow[k] ← Qtr[i][k] − (Htr[i][k] mod p) + ((j + Qtr[i][k]) mod p)
17                 similarity ← similarity + unsat_pc[qrow[k]]
18             if similarity ≥ th̄ then
19                 e[i · p + j] ← e[i · p + j] ⊕ 1
20                 for k = 0 to m − 1 do
21                     for h = 0 to dv − 1 do
22                         idx ← (Htr[qrow[k]/p][h] + (qrow[k] mod p)) mod p
23                         s'[idx] ← s'[idx] ⊕ 1
24     iter ← iter + 1
25 until s' ≠ 0  AND  iter < imax
26 if s' = 0 then
27     return e, true
28 return e, false
```

syndrome (line 10 in Algorithm 3.1.4). The threshold for the similarity is selected as the one paired to $\bar{w}$ in `LutS` (line 11 in Algorithm 3.1.4).

Table 3.1: Running times for key generation, encryption and decryption as a function of the chosen category and the number of circulant blocks n_0 on an AMD Ryzen 5 1600 CPU at 3.2 GHz.

Category	n_0	KeyGen (ms)	Encrypt (ms)	Decrypt (ms)	Total CPU time Ephemeral KEM (ms)
1	2	34.11 (±1.07)	2.11 (±0.08)	16.78 (±0.53)	52.99
	3	16.02 (±0.26)	2.15 (±0.17)	21.65 (±1.71)	39.81
	4	13.41 (±0.23)	2.42 (±0.08)	24.31 (±0.86)	40.14
2–3	2	142.71 (±1.52)	8.11 (±0.21)	48.23 (±2.93)	199.05
	3	76.74 (±0.78)	8.79 (±0.20)	49.15 (±2.20)	134.68
	4	54.93 (±0.84)	9.46 (±0.28)	46.16 (±2.03)	110.55
4–5	2	427.38 (±5.15)	23.00 (±0.33)	91.78 (±5.38)	542.16
	3	227.71 (±1.71)	24.85 (±0.37)	92.42 (±4.50)	344.99
	4	162.34 (±2.39)	26.30 (±0.53)	127.16 (±4.42)	315.80

3.2 Benchmarks on a NIST compliant platform

We provide the results of a set of execution time benchmarks performed on the reference implementation provided in the submission package. Currently, no platform specific optimizations are in place, thus we expect these results to be quite consistent across different platforms.

The results were obtained measuring the required time for key generation, encryption (key encapsulation) and decryption (key decapsulation) as a function of the chosen security category and number of circulant blocks n_0. The measurements reported are obtained as the average of 100 executions of the reference implementation compiled with `gcc` 6.3.0 from Debian 9 amd64. Given the NIST requirement on the reference computing platform (an Intel x86_64 CPU) we instructed `gcc` to employ the most basic instruction set among the ones fitting the description (`-march=nocona` option). The generated binaries were run on an AMD Ryzen 5 1600 CPU at 3.2 GHz, locking the frequency scaling to the top frequency.

Table 3.1 reports the running times measured employing the `clock_gettime` primitive, selecting the `CLOCK_PROCESS_CPUTIME_ID` as the timer of choice, obtaining the CPU time taken by the process. As it can be noticed, the most computationally demanding primitive is the key generation, which has more than 80% of its computation time taken by the execution of a single modular inverse in $\mathbb{F}_2[x]/\langle x^p+1\rangle$ required to obtain the value of $L_{n_0-1}^{-1}$. The encryption primitive is the fastest among all, and its computation time is substantially entirely devoted ($> 99\%$) to the $n_0 - 1$ polynomial multiplications performing the encryption. The decryption primitive computation is dominated by the Q-decoder computation ($> 95\%$ of the time), with a minimal portion taken by the n_0 modular multiplications which reconstruct L_{n_0-1} and the one to compute the private syndrome fed into the Q-decoder.

Considering the computational cost of performing a KEM with ephemeral keys, the most advantageous choice is to pick $n_0 = 4$ for any security level, although the computational savings are more significant when considering high-security parameter choices (Category 3 and Category 5).

Table 3.2 reports the sizes of both the keypairs and the encapsulated secrets for LEDAkem. In particular, regarding the size of the private keys of LEDAkem we report both the size of the stored private key, i.e. the size of the `rndPrivateMatricesSeed` extracted from the system TRNG, and the

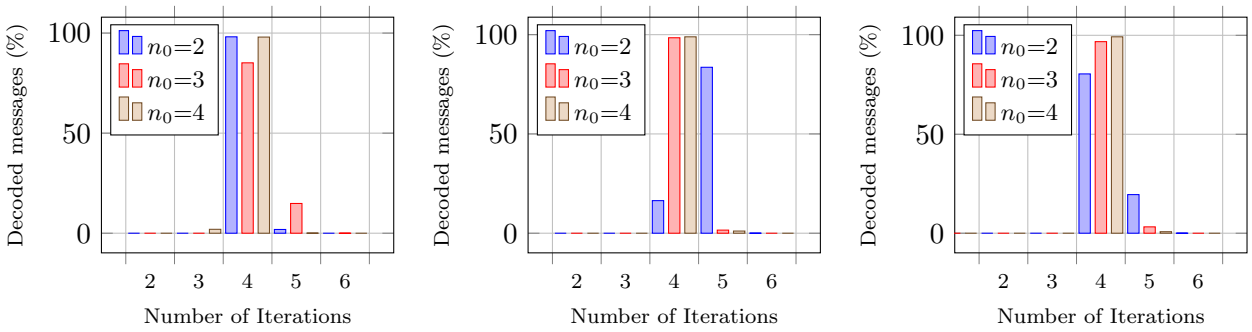
Table 3.2: Sizes of the keypair and encapsulated shared secret as a function of the chosen category and number of circulant blocks n_0

Category	n_0	Private Key Size (B)		Public Key size (B)	Shared secret size (B)	Enc secret size (B)
		At rest	In memory			
1	2	24	668	3,480	3,480	32
	3	24	844	4,688	2,344	32
	4	24	1,036	6,408	2,136	32
2–3	2	32	972	7,200	7,200	48
	3	32	1,196	10,384	5,192	48
	4	32	1,364	13,152	4,384	48
4–5	2	40	1,244	12,384	12,384	64
	3	40	1,548	18,016	9,008	64
	4	40	1,772	22,704	7,568	64

required amount of main memory to store the expanded key during the decryption phase. We note that the private key sizes are the minimum possible, as the `rndPrivateMatricesSeed` extracted from the system TRNG should be incompressible. We employ as a `rndPrivateMatricesSeed` size of 192, 256 and 320 bits for security categories 1, 3, and 5, respectively, to provide a simple hedging against multi key attacks, as suggested on the NIST forum, and in the NIST frequently asked questions section of the call. We note that, for a given security category, increasing the value of n_0 enlarges the public key, as it is constituted of $(n_0 - 1)p$ bits. This increase in the size of the public key represents a tradeoff with the decrease of the size of the ciphertext to be transmitted since it is only p bits long, and p decreases if a larger number of blocks is selected, for a fixed security category. The size of the derived encapsulated secret is at least 256 bits, in order to meet the requirement reported in the NIST call for proposals, Section 3, item 3.b. The shared secret is derived employing the SHA-3 hash function with a 256, 384 or 512 bits digest, so to match the requirements of categories 1, 3, and 5, respectively.

Possible optimizations. Starting from the platform-agnostic reference implementation provided in this submission, a number of optimizations can be applied to improve the running time of LEDAkem. First of all, implementing a sub-quadratic multiplication for the elements of $\mathbb{F}_2[x]/\langle x^p + 1\rangle$, for which the best candidate for the NIST reference platform appears to be the Toom-Cook method in either its Toom-3 or its Toom-4 variant [10], is expected to reduce the time required to compute the encryption primitive quite significantly. The optimal choice of the Toom-Cook variant will also be dependent on the availability on the underlying CPU of binary polynomial multiplication instructions, also known as carryless multiplications. Indeed, such instructions, present on both modern x84_64 and ARM ISAs provide significant speedups in single-precision binary polynomial multiplication.

Providing a specialized procedure for the modular inverse in $\mathbb{F}_2[x]/\langle x^p + 1\rangle$, where both the guaranteed invertibility of the element at hand, and the low weight nature of the modulus are taken into account is expected to provide a significant speedup to the key generation phase, which is dominated by a single instance of such computation. Finally, exploiting the presence of vector instructions to perform modular addition will also provide performance boosts, as it provides an effective way of exploiting the large amount of data parallelism present in the computational primitives employed.



(a) Category 1 (b) Category 2–3 (c) Category 4–5

Figure 3.1: Percentage of decoded messages as a function of the number of iterations taken to the Q-decoder to converge

3.3 Protection against side-channel attacks

The two most common side channels exploited to breach practical implementations of cryptosystems are the execution time of the primitive and the instantaneous power consumption during its computation. In particular, in [13], it was shown how a QC-LDPC code-based system can be broken by means of simple power analysis, exploiting the control-flow dependent differences of the decoding algorithm examined. Furthermore, [33] provides the first practical evidence of a side channel attack relying on differential power analysis to extract the positions of the ones in the private parity-check matrix, hence exploiting dataflow dependencies. However, linear error correcting codes are amenable to extremely efficient countermeasures against power consumption-based side channel attacks due to the linear nature of the operations involved in the decoding process. Indeed, the same [33] provided a simple, but effective, countermeasure against differential power analysis through adding a random codeword to the input vector before computing the syndrome. We note that employing ephemeral keys provides a natural layer of resistance against non-profiled power consumption side channel attacks. However, it may be possible to employ profiled side channel attack techniques to overcome this layer of protection, and against which the linearity of error correcting codes may be again exploited to devise efficient countermeasures.

Concerning execution time side channel information leakage, the main portion of the LEDAkem decryption algorithm which is not characterized by a constant execution time is decoding. Indeed, the number of iterations made by the decoder depends on the values being processed. Willing to provide a first quantitative estimate of the information leakage stemming by such a timing variation, Figure 3.1 reports the percentage of decoding actions taking a given number of iterations to perform a correct decode action during our DFR characterization campaign, i.e. over at least 10^8 decoding actions for each category/n_0 value pair. As it can be seen, the vast majority of decoding actions are completed in the same number of iterations. To provide a quantitative estimate of the information which may be obtained, we note that, considering the number of iterations required to perform a decode action as a random variable over the integers, and considering the frequencies divided by the total amount of decoding actions as a rough estimate of the actual probabilities, we obtain that the entropy of the random variables considered is between 0.21 and 0.01 bits per symbol, which is a quite limited amount of information. Nonetheless, it is possible to achieve a constant time decoding simply modifying the algorithm so that it always run for the maximum needed amount

of iterations to achieve the desired DFR. Such a choice completely eliminates the timing leakage, albeit trading it off for a performance penalty.

3.4 KAT values

Known answer tests generated for 100 runs of LEDAkem can be found in the `KAT` directory of the submission package. The naming convention of the `req/rsp` file pairs is the following:

`PQCkemKAT_<private_key_size>_<value_of_the_n0_parameter>.req`
`PQCkemKAT_<private_key_size>_<value_of_the_n0_parameter>.rsp`

In the following we report the SHA-2-256 digests of the KAT files, as obtainable via `sha256sum` or an analogous tool.

```
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_24_2.req
f7f6d1efb7ea980e20b7799b572a5306aedcee6c47e735b7b8cf0070ee97bfd8  PQCkemKAT_24_2.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_24_3.req
3da4e79b11f6791330718aca980ce6877ed8665c120a2dd6375e284e7d115af4  PQCkemKAT_24_3.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_24_4.req
39b1be9b7291ddf725cc664d5e2231b7e8a599c8ea1e4591adbd80413487b82a  PQCkemKAT_24_4.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_32_2.req
ba5eb32705e31a319aad44bf9269e08ab39ac06c3f8a19d3329efcf768aa7e3f  PQCkemKAT_32_2.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_32_3.req
8e87ad90886615deabb8876fc33fb1dec2127c376fdb9e987a9492b571fabe21  PQCkemKAT_32_3.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_32_4.req
13d6e728babc1d617dcf3674efb72a4f40dcef7dbbca290865046ec3a1df933f  PQCkemKAT_32_4.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_40_2.req
6a2b181fefc855afb634be6f108416115c8081faf377687efb8c5303f8d61dd0  PQCkemKAT_40_2.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_40_3.req
6dd8003901593707fab42bd1e0b9ce2875d9cdd9bd9f5e11d23e10770fc0a5f1  PQCkemKAT_40_3.rsp
36c27b6089b8910733a01fea1136469769b3ca3c35f2b375cfcc592f2112cfaa  PQCkemKAT_40_4.req
f2eacd9f6ed199d43a87e90066f613af7b16e725b7427fa241ad36d6a1bc48d6  PQCkemKAT_40_4.rsp
```

Chapter 4

Summary of advantages and limitations

+ Built on an NP-complete problem under reasonable assumptions.
+ Exploits improved BF decoders which are faster than classical BF decoders.
+ Compact keypairs (below 23 kiB at most), minimum size private keys.
+ Requires only addition and multiplication over $\mathbb{F}_2[x]$, and modular inverse over $\mathbb{F}_2[x]/\langle x^p+1\rangle$ besides single-precision integer operations.
+ Fully patent free, self contained, public domain codebase written in ANSI-C-99.
+ Easy to integrate in existing cryptographic libraries.
+ Particularly efficient to apply countermeasures against non-profiled power consumption and electromagnetic emissions side channel attacks.

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Statement by Each Submitter

I, Marco Baldi, of Università Politecnica delle Marche, Dipartimento di Ingegneria dell'Informazione (DII), Via Brecce Bianche 12, I-60131, Ancona, Italy, do hereby declare that the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem, is my own original work, or if submitted jointly with others, is the original work of the joint submitters. I further declare that I do not hold and do not intend to hold any patent or patent application with a claim which may cover the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem.

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I acknowledge that NIST will announce any selected cryptosystem(s) and proceed to publish the draft standards for public comment.

I do hereby agree to provide the statements required by Sections 2.D.2 and 2.D.3, below, for any patent or patent application identified to cover the practice of my cryptosystem, reference implementation or optimized implementations and the right to use such implementations for the purposes of the public review and evaluation process.

I acknowledge that, during the post-quantum algorithm evaluation process, NIST may remove my cryptosystem from consideration for standardization. If my cryptosystem (or the derived cryptosystem) is removed from consideration for standardization or withdrawn from consideration by all submitter(s) and owner(s), I understand that rights granted and assurances made under Sections 2.D.1, 2.D.2 and 2.D.3, including use rights of the reference and optimized implementations, may be withdrawn by the submitter(s) and owner(s), as appropriate.

Signed:

Title:

Date:

Place:

Statement by Reference/Optimized Implementations' Owner(s)

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Statement by Each Submitter

I, Alessandro Barenghi, of Politecnico di Milano, Dipartimento di Elettronica, Informazione e Bioingegneria (DEIB), Via G. Ponzio 34/5, I-20133, Milano, Italy, do hereby declare that the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem, is my own original work, or if submitted jointly with others, is the original work of the joint submitters. I further declare that I do not hold and do not intend to hold any patent or patent application with a claim which may cover the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem.

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Statement by Each Submitter

I, Franco Chiaraluce, of Università Politecnica delle Marche, Dipartimento di Ingegneria dell'Informazione (DII), Via Brecce Bianche 12, I-60131, Ancona, Italy, do hereby declare that the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem, is my own original work, or if submitted jointly with others, is the original work of the joint submitters. I further declare that I do not hold and do not intend to hold any patent or patent application with a claim which may cover the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem.

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Statement by Each Submitter

I, Gerardo Pelosi, of Politecnico di Milano, Dipartimento di Elettronica, Informazione e Bioingegneria (DEIB), Via G. Ponzio 34/5, I-20133, Milano, Italy, do hereby declare that the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem, is my own original work, or if submitted jointly with others, is the original work of the joint submitters. I further declare that I do not hold and do not intend to hold any patent or patent application with a claim which may cover the cryptosystem, reference implementation, or optimized implementations that I have submitted, known as LEDAkem.

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